MOJZA

A Level

Mathematics(P3)

9709/03 Difficult Question Compilation

BY TEAM MOJZA

CONTENTS

Year 2018

- m18_qp_32

Q1

Use the trapezium rule with three intervals to estimate the value of

$$\int_0^{\frac{1}{4}\pi} \sqrt{(1 - \tan x)}\, dx,$$

giving your answer correct to 3 decimal places. [3]

Q3

(i) Using the expansions of $\cos(3x + x)$ and $\cos(3x - x)$, show that

$$\tfrac{1}{2}(\cos 4x + \cos 2x) \equiv \cos 3x \cos x.$$ [3]

(ii) Hence show that $\int_{-\frac{1}{6}\pi}^{\frac{1}{6}\pi} \cos 3x \cos x\, dx = \frac{3}{8}\sqrt{3}$. [3]

Q4

The variables x and y satisfy the equation $y^n = Ax^3$, where n and A are constants. It is given that $y = 2.58$ when $x = 1.20$, and $y = 9.49$ when $x = 2.51$.

(i) Explain why the graph of $\ln y$ against $\ln x$ is a straight line. [2]

(ii) Find the values of n and A, giving your answers correct to 2 decimal places. [4]

Q7

(i) By sketching suitable graphs, show that the equation $e^{2x} = 6 + e^{-x}$ has exactly one real root. [2]

(ii) Verify by calculation that this root lies between 0.5 and 1. [2]

(iii) Show that if a sequence of values given by the iterative formula

$$x_{n+1} = \tfrac{1}{3}\ln(1 + 6e^{x_n})$$

converges, then it converges to the root of the equation in part **(i)**. [2]

(iv) Use this iterative formula to calculate the root correct to 3 decimal places. Give the result of each iteration to 5 decimal places. [3]

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Q3

A curve has equation $y = \dfrac{e^{3x}}{\tan \frac{1}{2}x}$. Find the x-coordinates of the stationary points of the curve in the interval $0 < x < \pi$. Give your answers correct to 3 decimal places. [6]

Q5

Let $I = \displaystyle\int_{\frac{1}{4}}^{\frac{3}{4}} \sqrt{\left(\frac{x}{1-x}\right)}\,dx$.

(i) Using the substitution $x = \cos^2\theta$, show that $I = \displaystyle\int_{\frac{1}{6}\pi}^{\frac{1}{3}\pi} 2\cos^2\theta\,d\theta$. [4]

(ii) Hence find the exact value of I. [4]

Q7

(i) Showing all working and without using a calculator, solve the equation $z^2 + (2\sqrt{6})z + 8 = 0$, giving your answers in the form $x + iy$, where x and y are real and exact. [3]

(ii) Sketch an Argand diagram showing the points representing the roots. [1]

(iii) The points representing the roots are A and B, and O is the origin. Find angle AOB. [3]

(iv) Prove that triangle AOB is equilateral. [1]

Q9

Let $f(x) = \dfrac{12x^2 + 4x - 1}{(x-1)(3x+2)}$.

(i) Express $f(x)$ in partial fractions. [5]

(ii) Hence obtain the expansion of $f(x)$ in ascending powers of x, up to and including the term in x^2. [5]

Q10

The point P has position vector $3\mathbf{i} - 2\mathbf{j} + \mathbf{k}$. The line l has equation $\mathbf{r} = 4\mathbf{i} + 2\mathbf{j} + 5\mathbf{k} + \mu(\mathbf{i} + 2\mathbf{j} + 3\mathbf{k})$.

(i) Find the length of the perpendicular from P to l, giving your answer correct to 3 significant figures. [5]

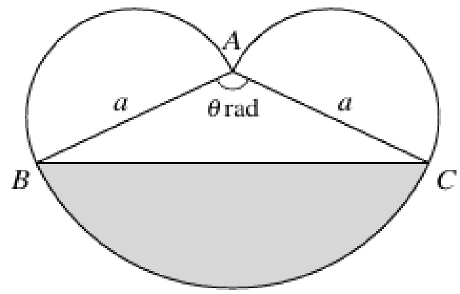
(ii) Find the equation of the plane containing l and P, giving your answer in the form $ax + by + cz = d$. [5]

- s18_qp_32

Q2

Showing all necessary working, solve the equation $\cot\theta + \cot(\theta + 45°) = 2$, for $0° < \theta < 180°$. [5]

Q6



The diagram shows a triangle ABC in which $AB = AC = a$ and angle $BAC = \theta$ radians. Semicircles are drawn outside the triangle with AB and AC as diameters. A circular arc with centre A joins B and C. The area of the shaded segment is equal to the sum of the areas of the semicircles.

(i) Show that $\theta = \frac{1}{2}\pi + \sin\theta$. [3]

(ii) Verify by calculation that θ lies between 2.2 and 2.4. [2]

(iii) Use an iterative formula based on the equation in part **(i)** to determine θ correct to 2 decimal places. Give the result of each iteration to 4 decimal places. [3]

Q10

Two lines l and m have equations $\mathbf{r} = 2\mathbf{i} - \mathbf{j} + \mathbf{k} + s(2\mathbf{i} + 3\mathbf{j} - \mathbf{k})$ and $\mathbf{r} = \mathbf{i} + 3\mathbf{j} + 4\mathbf{k} + t(\mathbf{i} + 2\mathbf{j} + \mathbf{k})$ respectively.

(i) Show that the lines are skew. [4]

A plane p is parallel to the lines l and m.

(ii) Find a vector that is normal to p. [3]

(iii) Given that p is equidistant from the lines l and m, find the equation of p. Give your answer in the form $ax + by + cz = d$. [3]

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Q5

(i) By first expanding $(\cos^2 x + \sin^2 x)^3$, or otherwise, show that

$$\cos^6 x + \sin^6 x = 1 - \tfrac{3}{4}\sin^2 2x. \qquad [4]$$

(ii) Hence solve the equation

$$\cos^6 x + \sin^6 x = \tfrac{2}{3},$$

for $0° < x < 180°$. [4]

Q8

The equation of a curve is $2x^3 - y^3 - 3xy^2 = 2a^3$, where a is a non-zero constant.

(i) Show that $\dfrac{dy}{dx} = \dfrac{2x^2 - y^2}{y^2 + 2xy}$. [4]

(ii) Find the coordinates of the two points on the curve at which the tangent is parallel to the y-axis. [5]

Q9

(a) Find the complex number z satisfying the equation

$$3z - iz^* = 1 + 5i,$$

where z^* denotes the complex conjugate of z. [4]

(b) On a sketch of an Argand diagram, shade the region whose points represent complex numbers z which satisfy both the inequalities $|z| \leqslant 3$ and $\operatorname{Im} z \geqslant 2$, where $\operatorname{Im} z$ denotes the imaginary part of z. Calculate the greatest value of $\arg z$ for points in this region. Give your answer in radians correct to 2 decimal places. [5]

Q10

The points A and B have position vectors $2\mathbf{i} + \mathbf{j} + 3\mathbf{k}$ and $4\mathbf{i} + \mathbf{j} + \mathbf{k}$ respectively. The line l has equation $\mathbf{r} = 4\mathbf{i} + 6\mathbf{j} + \mu(\mathbf{i} + 2\mathbf{j} - 2\mathbf{k})$.

(i) Show that l does not intersect the line passing through A and B. [5]

The point P, with parameter t, lies on l and is such that angle PAB is equal to $120°$.

(ii) Show that $3t^2 + 8t + 4 = 0$. Hence find the position vector of P. [6]

- w18_qp_31

Q3

(i) By sketching a suitable pair of graphs, show that the equation $x^3 = 3 - x$ has exactly one real root. [2]

Q4

The parametric equations of a curve are

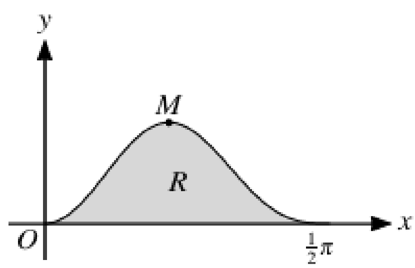
$$x = 2\sin\theta + \sin 2\theta, \quad y = 2\cos\theta + \cos 2\theta,$$

where $0 < \theta < \pi$.

(i) Obtain an expression for $\frac{dy}{dx}$ in terms of θ. [3]

(ii) Hence find the exact coordinates of the point on the curve at which the tangent is parallel to the y-axis. [4]

Q7



The diagram shows the curve $y = 5\sin^2 x \cos^3 x$ for $0 \leqslant x \leqslant \frac{1}{2}\pi$, and its maximum point M. The shaded region R is bounded by the curve and the x-axis.

(i) Find the x-coordinate of M, giving your answer correct to 3 decimal places. [5]

(ii) Using the substitution $u = \sin x$ and showing all necessary working, find the exact area of R. [4]

Q9

Let $f(x) = \frac{6x^2 + 8x + 9}{(2 - x)(3 + 2x)^2}$.

(i) Express $f(x)$ in partial fractions. [5]

(ii) Hence, showing all necessary working, show that $\int_{-1}^{0} f(x)\,dx = 1 + \frac{1}{2}\ln\left(\frac{3}{4}\right)$. [5]

- w18_qp_32

Q3

(i) Find $\int \frac{\ln x}{x^3}\,dx$. [3]

(ii) Hence show that $\int_1^2 \frac{\ln x}{x^3}\,dx = \frac{1}{16}(3 - \ln 4)$. [2]

Q4

Showing all necessary working, solve the equation

$$\frac{e^x + e^{-x}}{e^x + 1} = 4,$$

giving your answer correct to 3 decimal places. [5]

Q10

The line l has equation $\mathbf{r} = 5\mathbf{i} - 3\mathbf{j} - \mathbf{k} + \lambda(\mathbf{i} - 2\mathbf{j} + \mathbf{k})$. The plane p has equation

$$(\mathbf{r} - \mathbf{i} - 2\mathbf{j}).(3\mathbf{i} + \mathbf{j} + \mathbf{k}) = 0.$$

The line l intersects the plane p at the point A.

(i) Find the position vector of A. [3]

(ii) Calculate the acute angle between l and p. [4]

(iii) Find the equation of the line which lies in p and intersects l at right angles. [4]

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Q3

(i) Given that $\sin(\theta + 45°) + 2\cos(\theta + 60°) = 3\cos\theta$, find the exact value of $\tan\theta$ in a form involving surds. You need not simplify your answer. [4]

(ii) Hence solve the equation $\sin(\theta + 45°) + 2\cos(\theta + 60°) = 3\cos\theta$ for $0° < \theta < 360°$. [2]

Q5

The variables x and y satisfy the relation $\sin y = \tan x$, where $-\frac{1}{2}\pi < y < \frac{1}{2}\pi$. Show that

$$\frac{dy}{dx} = \frac{1}{\cos x\sqrt{(\cos 2x)}}.$$ [5]

Q6

The variables x and y satisfy the differential equation

$$\frac{dy}{dx} = ky^3e^{-x},$$

where k is a constant. It is given that $y = 1$ when $x = 0$, and that $y = \sqrt{e}$ when $x = 1$. Solve the differential equation, obtaining an expression for y in terms of x. [7]

Q7

(a) Showing all working and without using a calculator, solve the equation

$$(1 + i)z^2 - (4 + 3i)z + 5 + i = 0.$$

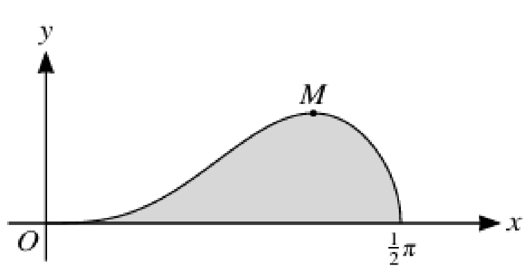
Give your answers in the form $x + iy$, where x and y are real. [6]

(b) The complex number u is given by

$$u = -1 - i.$$

On a sketch of an Argand diagram show the point representing u. Shade the region whose points represent complex numbers satisfying the inequalities $|z| < |z - 2i|$ and $\frac{1}{4}\pi < \arg(z - u) < \frac{1}{2}\pi$. [4]

Q10



The diagram shows the curve $y = \sin^3 x \sqrt{(\cos x)}$ for $0 \leqslant x \leqslant \frac{1}{2}\pi$, and its maximum point M.

(i) Using the substitution $u = \cos x$, find by integration the exact area of the shaded region bounded by the curve and the x-axis. [6]

(ii) Showing all your working, find the x-coordinate of M, giving your answer correct to 3 decimal places. [6]

- s19_qp_31

Q5

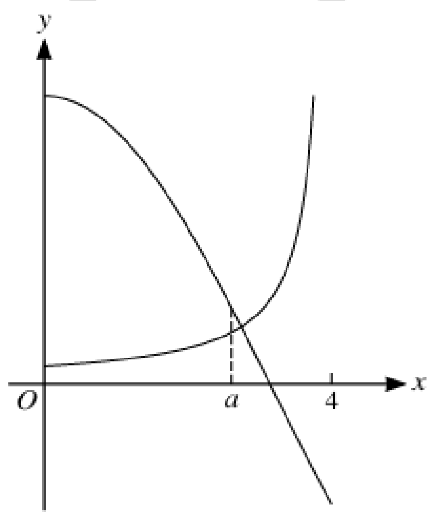
(i) Differentiate $\dfrac{1}{\sin^2 \theta}$ with respect to θ. [2]

(ii) The variables x and θ satisfy the differential equation

$$x \tan \theta \frac{dx}{d\theta} + \operatorname{cosec}^2 \theta = 0,$$

for $0 < \theta < \frac{1}{2}\pi$ and $x > 0$. It is given that $x = 4$ when $\theta = \frac{1}{6}\pi$. Solve the differential equation, obtaining an expression for x in terms of θ. [6]

Q7



The diagram shows the curves $y = 4\cos\frac{1}{2}x$ and $y = \dfrac{1}{4 - x}$, for $0 \leqslant x < 4$. When $x = a$, the tangents to the curves are perpendicular.

(i) Show that $a = 4 - \sqrt{(2\sin\frac{1}{2}a)}$. [4]

(ii) Verify by calculation that a lies between 2 and 3. [2]

(iii) Use an iterative formula based on the equation in part **(i)** to determine a correct to 3 decimal places. Give the result of each iteration to 5 decimal places. [3]

- s19_qp_32

Q5

Throughout this question the use of a calculator is not permitted.

It is given that the complex number $-1 + (\sqrt{3})i$ is a root of the equation

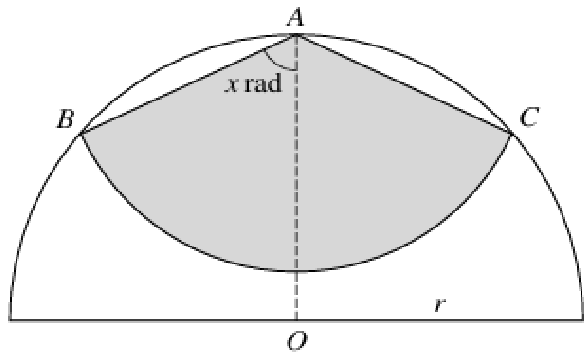
$$kx^3 + 5x^2 + 10x + 4 = 0,$$

where k is a real constant.

(i) Write down another root of the equation. [1]

(ii) Find the value of k and the third root of the equation. [6]

Q6



In the diagram, A is the mid-point of the semicircle with centre O and radius r. A circular arc with centre A meets the semicircle at B and C. The angle OAB is equal to x radians. The area of the shaded region bounded by AB, AC and the arc with centre A is equal to half the area of the semicircle.

(i) Use triangle OAB to show that $AB = 2r\cos x$. [1]

(ii) Hence show that $x = \cos^{-1}\sqrt{\left(\frac{\pi}{16x}\right)}$. [2]

(iii) Verify by calculation that x lies between 1 and 1.5. [2]

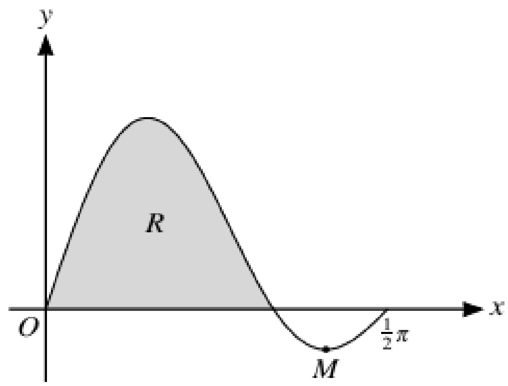
(iv) Use an iterative formula based on the equation in part **(ii)** to determine x correct to 3 decimal places. Give the result of each iteration to 5 decimal places. [3]

Q9

The points A and B have position vectors $\mathbf{i} + 2\mathbf{j} - \mathbf{k}$ and $3\mathbf{i} + \mathbf{j} + \mathbf{k}$ respectively. The line l has equation $\mathbf{r} = 2\mathbf{i} + \mathbf{j} + \mathbf{k} + \mu(\mathbf{i} + \mathbf{j} + 2\mathbf{k})$.

(i) Show that l does not intersect the line passing through A and B. [5]

Q10



The diagram shows the curve $y = \sin 3x \cos x$ for $0 \leqslant x \leqslant \frac{1}{2}\pi$ and its minimum point M. The shaded region R is bounded by the curve and the x-axis.

(i) By expanding $\sin(3x + x)$ and $\sin(3x - x)$ show that

$$\sin 3x \cos x = \tfrac{1}{2}(\sin 4x + \sin 2x).$$ [3]

(ii) Using the result of part **(i)** and showing all necessary working, find the exact area of the region R. [4]

(iii) Using the result of part **(i)**, express $\frac{dy}{dx}$ in terms of $\cos 2x$ and hence find the x-coordinate of M, giving your answer correct to 2 decimal places. [5]

- s19_qp_33

Q2

Show that $\int_0^{\frac{1}{4}\pi} x^2 \cos 2x \, dx = \frac{1}{32}(\pi^2 - 8)$. [5]

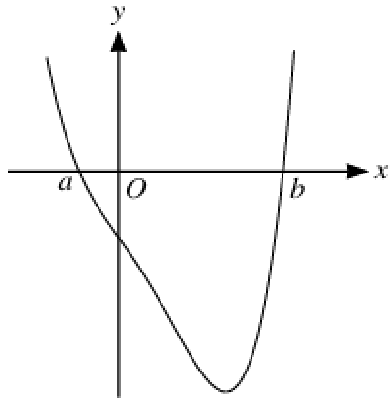
Q4

The equation of a curve is $y = \dfrac{1 + e^{-x}}{1 - e^{-x}}$, for $x > 0$.

(i) Show that $\frac{dy}{dx}$ is always negative. [3]

(ii) The gradient of the curve is equal to -1 when $x = a$. Show that a satisfies the equation $e^{2a} - 4e^a + 1 = 0$. Hence find the exact value of a. [4]

Q6



The diagram shows the curve $y = x^4 - 2x^3 - 7x - 6$. The curve intersects the x-axis at the points $(a, 0)$ and $(b, 0)$, where $a < b$. It is given that b is an integer.

(i) Find the value of b. [1]

(ii) Hence show that a satisfies the equation $a = -\frac{1}{3}(2 + a^2 + a^3)$. [4]

(iii) Use an iterative formula based on the equation in part **(ii)** to determine a correct to 3 decimal places. Give the result of each iteration to 5 decimal places. [3]

- w19_qp_31

Q4

The number of insects in a population t weeks after the start of observations is denoted by N. The population is decreasing at a rate proportional to $Ne^{-0.02t}$. The variables N and t are treated as continuous, and it is given that when $t = 0$, $N = 1000$ and $\frac{dN}{dt} = -10$.

(i) Show that N and t satisfy the differential equation

$$\frac{dN}{dt} = -0.01e^{-0.02t}N.$$ [1]

(ii) Solve the differential equation and find the value of t when $N = 800$. [6]

(iii) State what happens to the value of N as t becomes large. [1]

Q5

The curve with equation $y = e^{-2x}\ln(x - 1)$ has a stationary point when $x = p$.

(i) Show that p satisfies the equation $x = 1 + \exp\left(\frac{1}{2(x-1)}\right)$, where $\exp(x)$ denotes e^x. [3]

(ii) Verify by calculation that p lies between 2.2 and 2.6. [2]

(iii) Use an iterative formula based on the equation in part **(i)** to determine p correct to 2 decimal places. Give the result of each iteration to 4 decimal places. [3]

Q10

(a) The complex number u is given by $u = -3 - (2\sqrt{10})\text{i}$. Showing all necessary working and without using a calculator, find the square roots of u. Give your answers in the form $a + \text{i}b$, where the numbers a and b are real and exact. [5]

(b) On a sketch of an Argand diagram shade the region whose points represent complex numbers z satisfying the inequalities $|z - 3 - \text{i}| \leqslant 3$, $\arg z \geqslant \frac{1}{4}\pi$ and $\text{Im}\, z \geqslant 2$, where $\text{Im}\, z$ denotes the imaginary part of the complex number z. [5]

- w19_qp_32

Q1

Solve the equation $5\ln(4 - 3^x) = 6$. Show all necessary working and give the answer correct to 3 decimal places. [3]

Q3

The polynomial $x^4 + 3x^3 + ax + b$, where a and b are constants, is denoted by $\text{p}(x)$. When $\text{p}(x)$ is divided by $x^2 + x - 1$ the remainder is $2x + 3$. Find the values of a and b. [5]

Q4

(i) Express $(\sqrt{6})\sin x + \cos x$ in the form $R\sin(x + \alpha)$, where $R > 0$ and $0° < \alpha < 90°$. State the exact value of R and give α correct to 3 decimal places. [3]

(ii) Hence solve the equation $(\sqrt{6})\sin 2\theta + \cos 2\theta = 2$, for $0° < \theta < 180°$. [4]

Q5

The equation of a curve is $2x^2y - xy^2 = a^3$, where a is a positive constant. Show that there is only one point on the curve at which the tangent is parallel to the x-axis and find the y-coordinate of this point. [7]

Q7

(a) Find the complex number z satisfying the equation

$$z + \frac{\text{i}z}{z^*} - 2 = 0,$$

where z^* denotes the complex conjugate of z. Give your answer in the form $x + \text{i}y$, where x and y are real. [5]

(b) **(i)** On a single Argand diagram sketch the loci given by the equations $|z - 2\text{i}| = 2$ and $\text{Im}\, z = 3$, where $\text{Im}\, z$ denotes the imaginary part of z. [2]

(ii) In the first quadrant the two loci intersect at the point P. Find the exact argument of the complex number represented by P. [2]

- w19_qp_33

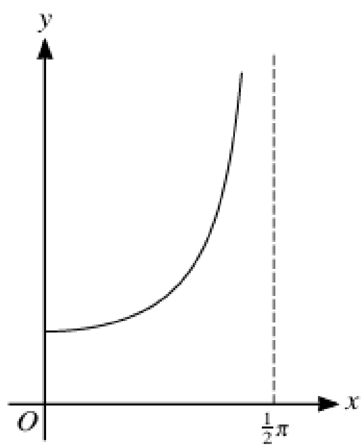
Q3

Showing all necessary working, solve the equation $\dfrac{3^{2x}+3^{-x}}{3^{2x}-3^{-x}}=4$. Give your answer correct to 3 decimal places. [4]

Q5

(i) By sketching a suitable pair of graphs, show that the equation $\ln(x+2)=4e^{-x}$ has exactly one real root. [2]

Q8



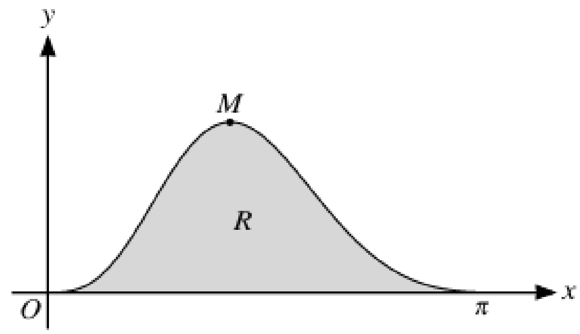
The diagram shows the graph of $y=\sec x$ for $0 \leqslant x < \frac{1}{2}\pi$.

(i) Use the trapezium rule with 2 intervals to estimate the value of $\displaystyle\int_0^{1.2} \sec x \, dx$, giving your answer correct to 2 decimal places. [3]

(ii) Explain, with reference to the diagram, whether the trapezium rule gives an overestimate or an underestimate of the true value of the integral in part **(i)**. [1]

(iii) P is the point on the part of the curve $y=\sec x$ for $0 \leqslant x < \frac{1}{2}\pi$ at which the gradient is 2. By first differentiating $\dfrac{1}{\cos x}$, find the x-coordinate of P, giving your answer correct to 3 decimal places. [6]

Q10



The diagram shows the graph of $y = e^{\cos x} \sin^3 x$ for $0 \leqslant x \leqslant \pi$, and its maximum point M. The shaded region R is bounded by the curve and the x-axis.

(i) Find the x-coordinate of M. Show all necessary working and give your answer correct to 2 decimal places. [5]

(ii) By first using the substitution $u = \cos x$, find the exact value of the area of R. [7]

Year 2020

- m20_qp_32

Q1

(a) Sketch the graph of $y = |x - 2|$. [1]

(b) Solve the inequality $|x - 2| < 3x - 4$. [3]

Q3

(a) By sketching a suitable pair of graphs, show that the equation $\sec x = 2 - \frac{1}{2}x$ has exactly one root in the interval $0 \leqslant x < \frac{1}{2}\pi$. [2]

(b) Verify by calculation that this root lies between 0.8 and 1. [2]

(c) Use the iterative formula $x_{n+1} = \cos^{-1}\left(\frac{2}{4 - x_n}\right)$ to determine the root correct to 2 decimal places. Give the result of each iteration to 4 decimal places. [3]

Q6

The variables x and y satisfy the differential equation

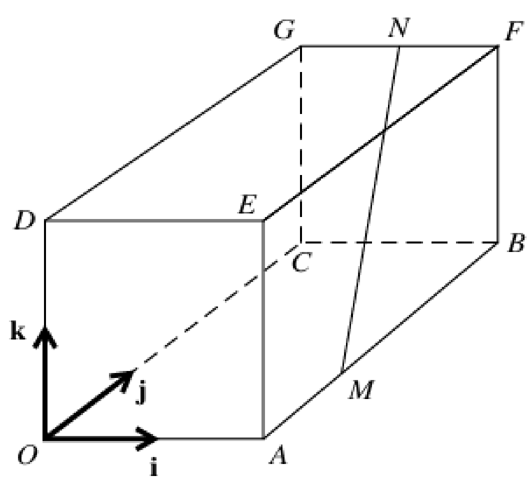
$$\frac{dy}{dx} = \frac{1 + 4y^2}{e^x}.$$

It is given that $y = 0$ when $x = 1$.

(a) Solve the differential equation, obtaining an expression for y in terms of x. [7]

(b) State what happens to the value of y as x tends to infinity. [1]

Q8



In the diagram, $OABCDEFG$ is a cuboid in which $OA = 2$ units, $OC = 3$ units and $OD = 2$ units. Unit vectors $\mathbf{i}$, $\mathbf{j}$ and $\mathbf{k}$ are parallel to OA, OC and OD respectively. The point M on AB is such that $MB = 2AM$. The midpoint of FG is N.

(a) Express the vectors $\overrightarrow{OM}$ and $\overrightarrow{MN}$ in terms of $\mathbf{i}$, $\mathbf{j}$ and $\mathbf{k}$. [3]

(b) Find a vector equation for the line through M and N. [2]

Q10

(a) The complex numbers v and w satisfy the equations

$$v + \mathrm{i}w = 5 \qquad \text{and} \qquad (1 + 2\mathrm{i})v - w = 3\mathrm{i}.$$

Solve the equations for v and w, giving your answers in the form $x + \mathrm{i}y$, where x and y are real. [6]

- w20_qp_31

Q5

(a) By sketching a suitable pair of graphs, show that the equation $\operatorname{cosec} x = 1 + \mathrm{e}^{-\frac{1}{2}x}$ has exactly two roots in the interval $0 < x < \pi$. [2]

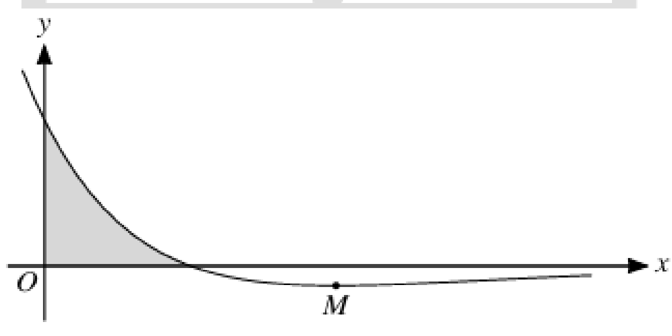
Q9

Let $\mathrm{f}(x) = \dfrac{8 + 5x + 12x^2}{(1 - x)(2 + 3x)^2}$.

(a) Express $\mathrm{f}(x)$ in partial fractions. [5]

(b) Hence obtain the expansion of $\mathrm{f}(x)$ in ascending powers of x, up to and including the term in x^2. [5]

Q10



The diagram shows the curve $y = (2 - x)\mathrm{e}^{-\frac{1}{2}x}$, and its minimum point M.

(a) Find the exact coordinates of M. [5]

(b) Find the area of the shaded region bounded by the curve and the axes. Give your answer in terms of e. [5]

MOJZA

Q11

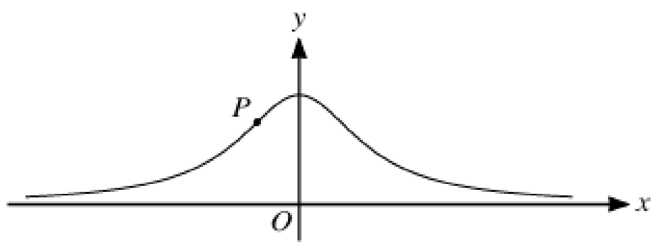
Two lines have equations $\mathbf{r} = \mathbf{i} + 2\mathbf{j} + \mathbf{k} + \lambda(a\mathbf{i} + 2\mathbf{j} - \mathbf{k})$ and $\mathbf{r} = 2\mathbf{i} + \mathbf{j} - \mathbf{k} + \mu(2\mathbf{i} - \mathbf{j} + \mathbf{k})$, where a is a constant.

(a) Given that the two lines intersect, find the value of a and the position vector of the point of intersection. [5]

(b) Given instead that the acute angle between the directions of the two lines is $\cos^{-1}\left(\frac{1}{6}\right)$, find the two possible values of a. [6]

- w20_qp_32

Q5



The diagram shows the curve with parametric equations

$$x = \tan\theta, \quad y = \cos^2\theta,$$

for $-\frac{1}{2}\pi < \theta < \frac{1}{2}\pi$.

(a) Show that the gradient of the curve at the point with parameter θ is $-2\sin\theta\cos^3\theta$. [3]

The gradient of the curve has its maximum value at the point P.

(b) Find the exact value of the x-coordinate of P. [4]

Q6

The complex number u is defined by

$$u = \frac{7 + \mathrm{i}}{1 - \mathrm{i}}.$$

(a) Express u in the form $x + \mathrm{i}y$, where x and y are real. [3]

(b) Show on a sketch of an Argand diagram the points A, B and C representing u, $7 + \mathrm{i}$ and $1 - \mathrm{i}$ respectively. [2]

(c) By considering the arguments of $7 + \mathrm{i}$ and $1 - \mathrm{i}$, show that

$$\tan^{-1}\left(\tfrac{4}{3}\right) = \tan^{-1}\left(\tfrac{1}{7}\right) + \tfrac{1}{4}\pi.$$ [3]

Q7

The variables x and t satisfy the differential equation

$$e^{3t}\frac{dx}{dt} = \cos^2 2x,$$

for $t \geqslant 0$. It is given that $x = 0$ when $t = 0$.

(a) Solve the differential equation and obtain an expression for x in terms of t. [7]

(b) State what happens to the value of x when t tends to infinity. [1]

Q9

Let $f(x) = \dfrac{7x + 18}{(3x + 2)(x^2 + 4)}$.

(a) Express $f(x)$ in partial fractions. [5]

(b) Hence find the exact value of $\int_0^2 f(x)\,dx$. [6]

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Q6

Let $f(x) = \dfrac{5a}{(2x-a)(3a-x)}$, where a is a positive constant.

(a) Express $f(x)$ in partial fractions. [3]

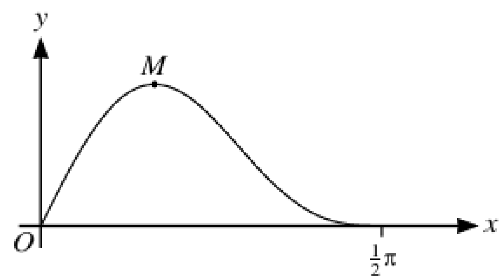
(b) Hence show that $\displaystyle\int_a^{2a} f(x)\,dx = \ln 6$. [4]

Q7

Two lines have equations $\mathbf{r} = \begin{pmatrix} 1 \\ 3 \\ 2 \end{pmatrix} + s\begin{pmatrix} 2 \\ -1 \\ 3 \end{pmatrix}$ and $\mathbf{r} = \begin{pmatrix} 2 \\ 1 \\ 4 \end{pmatrix} + t\begin{pmatrix} 1 \\ -1 \\ 4 \end{pmatrix}$.

(a) Show that the lines are skew. [5]

Q10



The diagram shows the curve $y = \sin 2x \cos^2 x$ for $0 \leqslant x \leqslant \frac{1}{2}\pi$, and its maximum point M.

(a) Using the substitution $u = \sin x$, find the exact area of the region bounded by the curve and the x-axis. [5]

(b) Find the exact x-coordinate of M. [6]

- s21_qp_31

Q2

Find the real root of the equation $\dfrac{2e^x + e^{-x}}{2 + e^x} = 3$, giving your answer correct to 3 decimal places. Your working should show clearly that the equation has only one real root. [5]

Q5

(a) Solve the equation $z^2 - 2piz - q = 0$, where p and q are real constants. [2]

MOJZA

In an Argand diagram with origin O, the roots of this equation are represented by the distinct points A and B.

(b) Given that A and B lie on the imaginary axis, find a relation between p and q. [2]

(c) Given instead that triangle OAB is equilateral, express q in terms of p. [3]

- s21_qp_32

Q3

The variables x and y satisfy the equation $x = A(3^{-y})$, where A is a constant.

(a) Explain why the graph of y against $\ln x$ is a straight line and state the exact value of the gradient of the line. [3]

It is given that the line intersects the y-axis at the point where $y = 1.3$.

(b) Calculate the value of A, giving your answer correct to 2 decimal places. [2]

Q4

Using integration by parts, find the exact value of $\int_0^2 \tan^{-1}\left(\frac{1}{2}x\right) \mathrm{d}x$. [5]

Q5

The complex number u is given by $u = 10 - 4\sqrt{6}\mathrm{i}$.

Find the two square roots of u, giving your answers in the form $a + \mathrm{i}b$, where a and b are real and exact. [5]

Q11

With respect to the origin O, the points A and B have position vectors given by $\overrightarrow{OA} = 2\mathbf{i} - \mathbf{j}$ and $\overrightarrow{OB} = \mathbf{j} - 2\mathbf{k}$.

(a) Show that $OA = OB$ and use a scalar product to calculate angle AOB in degrees. [4]

The midpoint of AB is M. The point P on the line through O and M is such that $PA : OA = \sqrt{7} : 1$.

(b) Find the possible position vectors of P. [6]

- s21_qp_33

Q3

The parametric equations of a curve are

$$x = t + \ln(t + 2), \qquad y = (t - 1)\mathrm{e}^{-2t},$$

where $t > -2$.

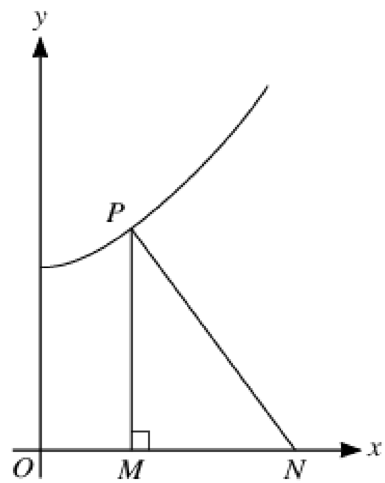
(a) Express $\frac{\mathrm{d}y}{\mathrm{d}x}$ in terms of t, simplifying your answer. [5]

(b) Find the exact y-coordinate of the stationary point of the curve. [2]

Q6

(a) By sketching a suitable pair of graphs, show that the equation $\cot \frac{1}{2}x = 1 + e^{-x}$ has exactly one root in the interval $0 < x \leqslant \pi$. [2]

Q7



For the curve shown in the diagram, the normal to the curve at the point P with coordinates (x, y) meets the x-axis at N. The point M is the foot of the perpendicular from P to the x-axis.

The curve is such that for all values of x in the interval $0 \leqslant x < \frac{1}{2}\pi$, the area of triangle PMN is equal to $\tan x$.

(a) **(i)** Show that $\dfrac{MN}{y} = \dfrac{dy}{dx}$. [1]

(ii) Hence show that x and y satisfy the differential equation $\frac{1}{2}y^2 \dfrac{dy}{dx} = \tan x$. [2]

- w21_qp_31

Q2

(a) Express $5 \sin x - 3 \cos x$ in the form $R \sin(x - \alpha)$, where $R > 0$ and $0 < \alpha < \frac{1}{2}\pi$. Give the exact value of R and give α correct to 2 decimal places. [3]

(b) Hence state the greatest and least possible values of $(5 \sin x - 3 \cos x)^2$. [2]

Q7

(a) Given that $y = \ln(\ln x)$, show that

$$\frac{dy}{dx} = \frac{1}{x \ln x}.$$ [1]

The variables x and t satisfy the differential equation

$$x \ln x + t\frac{dx}{dt} = 0.$$

It is given that $x = e$ when $t = 2$.

(b) Solve the differential equation obtaining an expression for x in terms of t, simplifying your answer. [7]

(c) Hence state what happens to the value of x as t tends to infinity. [1]

Q8

The constant a is such that $\int_1^a \frac{\ln x}{\sqrt{x}}\,dx = 6$.

(a) Show that $a = \exp\left(\frac{1}{\sqrt{a}} + 2\right)$. [5]

[$\exp(x)$ is an alternative notation for e^x.]

(b) Verify by calculation that a lies between 9 and 11. [2]

(c) Use an iterative formula based on the equation in part **(a)** to determine a correct to 2 decimal places. Give the result of each iteration to 4 decimal places. [3]

Q10

The complex number $1 + 2i$ is denoted by u. The polynomial $2x^3 + ax^2 + 4x + b$, where a and b are real constants, is denoted by $p(x)$. It is given that u is a root of the equation $p(x) = 0$.

(a) Find the values of a and b. [4]

(b) State a second complex root of this equation. [1]

(c) Find the real factors of $p(x)$. [2]

(d) **(i)** On a sketch of an Argand diagram, shade the region whose points represent complex numbers z satisfying the inequalities $|z - u| \leqslant \sqrt{5}$ and $\arg z \leqslant \frac{1}{4}\pi$. [4]

(ii) Find the least value of $\text{Im}\, z$ for points in the shaded region. Give your answer in an exact form. [1]

- w21_qp_32

Q3

(a) Given the complex numbers $u = a + ib$ and $w = c + id$, where a, b, c and d are real, prove that $(u + w)^* = u^* + w^*$. [2]

(b) Solve the equation $(z + 2 + i)^* + (2 + i)z = 0$, giving your answer in the form $x + iy$ where x and y are real. [4]

Q5

(a) On a sketch of an Argand diagram, shade the region whose points represent complex numbers z satisfying the inequalities $|z - 3 - 2\mathrm{i}| \leqslant 1$ and $\operatorname{Im} z \geqslant 2$. [4]

(b) Find the greatest value of $\arg z$ for points in the shaded region, giving your answer in degrees. [3]

Q8

(a) By first expanding $(\cos^2\theta + \sin^2\theta)^2$, show that

$$\cos^4\theta + \sin^4\theta \equiv 1 - \tfrac{1}{2}\sin^2 2\theta.$$ [3]

(b) Hence solve the equation

$$\cos^4\theta + \sin^4\theta = \tfrac{5}{9},$$

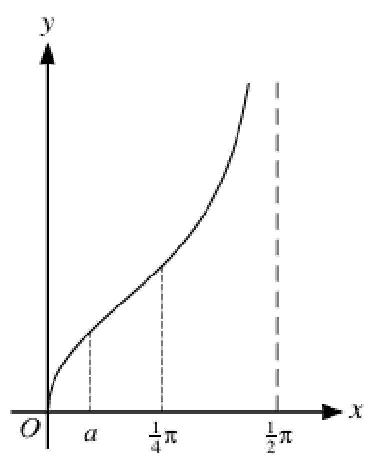
for $0° < \theta < 180°$. [4]

Q11

The equation of a curve is $y = \sqrt{\tan x}$, for $0 \leqslant x < \frac{1}{2}\pi$.

(a) Express $\frac{\mathrm{d}y}{\mathrm{d}x}$ in terms of $\tan x$, and verify that $\frac{\mathrm{d}y}{\mathrm{d}x} = 1$ when $x = \frac{1}{4}\pi$. [4]

The value of $\frac{\mathrm{d}y}{\mathrm{d}x}$ is also 1 at another point on the curve where $x = a$, as shown in the diagram.



(b) Show that $t^3 + t^2 + 3t - 1 = 0$, where $t = \tan a$. [4]

(c) Use the iterative formula

$$a_{n+1} = \tan^{-1}\left(\tfrac{1}{3}(1 - \tan^2 a_n - \tan^3 a_n)\right)$$

to determine a correct to 2 decimal places, giving the result of each iteration to 4 decimal places. [3]

- w21_qp_33

Q3

Solve the equation $4^{x-2} = 4^x - 4^2$, giving your answer correct to 3 decimal places. [4]

Q6

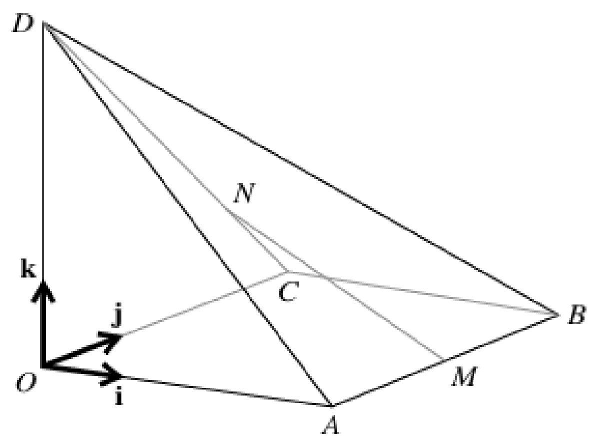
(a) By first expanding $\cos(x - 60°)$, show that the expression

$$2\cos(x - 60°) + \cos x$$

can be written in the form $R\cos(x - \alpha)$, where $R > 0$ and $0° < \alpha < 90°$. Give the exact value of R and the value of α correct to 2 decimal places. [5]

(b) Hence find the value of x in the interval $0° < x < 360°$ for which $2\cos(x - 60°) + \cos x$ takes its least possible value. [2]

Q8



In the diagram, $OABCD$ is a pyramid with vertex D. The horizontal base $OABC$ is a square of side 4 units. The edge OD is vertical and $OD = 4$ units. The unit vectors **i**, **j** and **k** are parallel to OA, OC and OD respectively.

The midpoint of AB is M and the point N on CD is such that $DN = 3NC$.

(a) Find a vector equation for the line through M and N. [5]

(b) Show that the length of the perpendicular from O to MN is $\frac{1}{3}\sqrt{82}$. [4]

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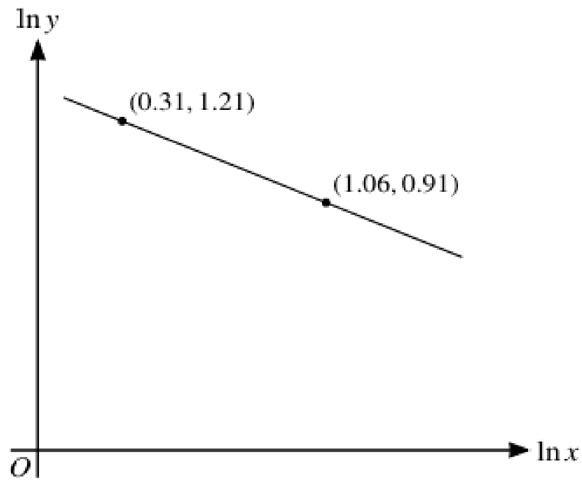
Q2

On a sketch of an Argand diagram, shade the region whose points represent complex numbers z satisfying the inequalities $|z + 2 - 3\mathrm{i}| \leqslant 2$ and $\arg z \leqslant \frac{3}{4}\pi$. [4]

Q3



The variables x and y satisfy the equation $x^n y^2 = C$, where n and C are constants. The graph of $\ln y$ against $\ln x$ is a straight line passing through the points (0.31, 1.21) and (1.06, 0.91), as shown in the diagram.

Find the value of n and find the value of C correct to 2 decimal places. [5]

Q4

The parametric equations of a curve are

$$x = 1 - \cos\theta, \qquad y = \cos\theta - \frac{1}{4}\cos 2\theta.$$

Show that $\dfrac{\mathrm{d}y}{\mathrm{d}x} = -2\sin^2\left(\frac{1}{2}\theta\right)$. [5]

Q7

(a) By sketching a suitable pair of graphs, show that the equation $4 - x^2 = \sec\frac{1}{2}x$ has exactly one root in the interval $0 \leqslant x < \pi$. [2]

(b) Verify by calculation that this root lies between 1 and 2. [2]

(c) Use the iterative formula $x_{n+1} = \sqrt{4 - \sec\frac{1}{2}x_n}$ to determine the root correct to 2 decimal places. Give the result of each iteration to 4 decimal places. [3]

- s22_qp_31

Q2

(a) Expand $(2-x^2)^{-2}$ in ascending powers of x, up to and including the term in x^4, simplifying the coefficients. [4]

(b) State the set of values of x for which the expansion is valid. [1]

Q5

The polynomial $ax^3 - 10x^2 + bx + 8$, where a and b are constants, is denoted by $p(x)$. It is given that $(x-2)$ is a factor of both $p(x)$ and $p'(x)$.

(a) Find the values of a and b. [5]

(b) When a and b have these values, factorise $p(x)$ completely. [3]

Q6

Let $I = \int_0^3 \frac{27}{(9+x^2)^2}\,dx$.

(a) Using the substitution $x = 3\tan\theta$, show that $I = \int_0^{\frac{1}{4}\pi} \cos^2\theta\, d\theta$. [4]

(b) Hence find the exact value of I. [4]

Q8

The equation of a curve is $x^3 + y^3 + 2xy + 8 = 0$.

(a) Express $\frac{dy}{dx}$ in terms of x and y. [4]

The tangent to the curve at the point where $x = 0$ and the tangent at the point where $y = 0$ intersect at the acute angle α.

(b) Find the exact value of $\tan\alpha$. [5]

- s22_qp_32

Q4

The equation of a curve is $y = \cos^3 x\sqrt{\sin x}$. It is given that the curve has one stationary point in the interval $0 < x < \frac{1}{2}\pi$.

Find the x-coordinate of this stationary point, giving your answer correct to 3 significant figures. [6]

Q5

(a) By sketching a suitable pair of graphs, show that the equation $\ln x = 3x - x^2$ has one real root. [2]

Q9

The lines l and m have vector equations

$$\mathbf{r} = -\mathbf{i} + 3\mathbf{j} + 4\mathbf{k} + \lambda(2\mathbf{i} - \mathbf{j} - \mathbf{k}) \quad \text{and} \quad \mathbf{r} = 5\mathbf{i} + 4\mathbf{j} + 3\mathbf{k} + \mu(a\mathbf{i} + b\mathbf{j} + \mathbf{k})$$

respectively, where a and b are constants.

(a) Given that l and m intersect, show that $2b - a = 4$. [4]

- s22_qp_33

Q3

(a) Show that the equation $\log_3(2x + 1) = 1 + 2\log_3(x - 1)$ can be written as a quadratic equation in x. [3]

(b) Hence solve the equation $\log_3(4y + 1) = 1 + 2\log_3(2y - 1)$, giving your answer correct to 2 decimal places. [2]

Q8

At time t days after the start of observations, the number of insects in a population is N. The variation in the number of insects is modelled by a differential equation of the form $\frac{dN}{dt} = kN^{\frac{3}{2}}\cos 0.02t$, where k is a constant and N is a continuous variable. It is given that when $t = 0$, $N = 100$.

(a) Solve the differential equation, obtaining a relation between N, k and t. [5]

(b) Given also that $N = 625$ when $t = 50$, find the value of k. [2]

(c) Obtain an expression for N in terms of t, and find the greatest value of N predicted by this model. [2]

- w22_qp_31

Q5

The complex numbers u and w are defined by $u = 2e^{\frac{1}{4}\pi i}$ and $w = 3e^{\frac{1}{3}\pi i}$.

(a) Find $\frac{u^2}{w}$, giving your answer in the form $re^{i\theta}$, where $r > 0$ and $-\pi < \theta \leqslant \pi$. Give the exact values of r and θ. [3]

(b) State the least positive integer n such that both $\operatorname{Im} w^n = 0$ and $\operatorname{Re} w^n > 0$. [1]

Q7

The equation of a curve is $y = \frac{x}{\cos^2 x}$, for $0 \leqslant x < \frac{1}{2}\pi$. At the point where $x = a$, the tangent to the curve has gradient equal to 12.

(a) Show that $a = \cos^{-1}\left(\sqrt[3]{\frac{\cos a + 2a\sin a}{12}}\right)$. [3]

(b) Verify by calculation that a lies between 0.9 and 1. [2]

(c) Use an iterative formula based on the equation in part **(a)** to determine a correct to 2 decimal places. Give the result of each iteration to 4 decimal places. [3]

Q8

In a certain chemical reaction the amount, x grams, of a substance is increasing. The differential equation satisfied by x and t, the time in seconds since the reaction began, is

$$\frac{dx}{dt} = kxe^{-0.1t},$$

where k is a positive constant. It is given that $x = 20$ at the start of the reaction.

(a) Solve the differential equation, obtaining a relation between x, t and k. [5]

(b) Given that $x = 40$ when $t = 10$, find the value of k and find the value approached by x as t becomes large. [3]

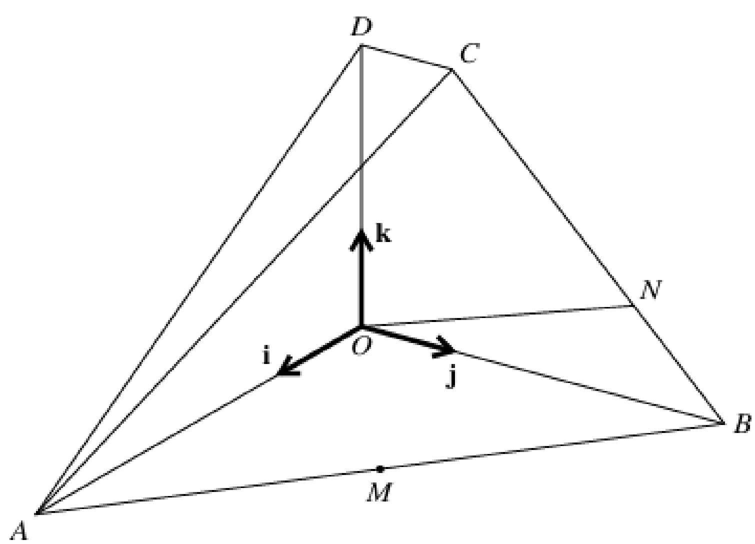
Q10

Let $f(x) = \dfrac{2x^2 + 7x + 8}{(1 + x)(2 + x)^2}$.

(a) Express $f(x)$ in partial fractions. [5]

(b) Hence obtain the expansion of $f(x)$ in ascending powers of x, up to and including the term in x^2. [5]

Q11



In the diagram, $OABCD$ is a solid figure in which $OA = OB = 4$ units and $OD = 3$ units. The edge OD is vertical, DC is parallel to OB and $DC = 1$ unit. The base, OAB, is horizontal and angle $AOB = 90°$. Unit vectors **i**, **j** and **k** are parallel to OA, OB and OD respectively. The midpoint of AB is M and the point N on BC is such that $CN = 2NB$.

(a) Express vectors $\overrightarrow{MD}$ and $\overrightarrow{ON}$ in terms of **i**, **j** and **k**. [4]

(b) Calculate the angle in degrees between the directions of $\overrightarrow{MD}$ and $\overrightarrow{ON}$. [3]

(c) Show that the length of the perpendicular from M to ON is $\sqrt{\frac{22}{5}}$. [4]

- w22_qp_32

Q1

Solve the equation $2^{3x-1} = 5(3^{1-x})$. Give your answer in the form $\frac{\ln a}{\ln b}$ where a and b are integers. [4]

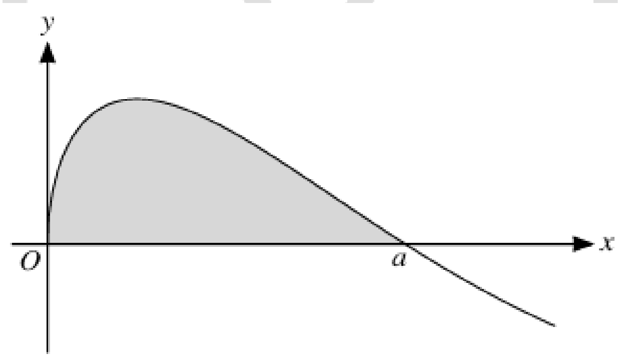
Q2

The polynomial $2x^3 - x^2 + a$, where a is a constant, is denoted by $p(x)$. It is given that $(2x + 3)$ is a factor of $p(x)$.

(a) Find the value of a. [2]

(b) When a has this value, solve the inequality $p(x) < 0$. [4]

Q8



The diagram shows part of the curve $y = \sin\sqrt{x}$. This part of the curve intersects the x-axis at the point where $x = a$.

(a) State the exact value of a. [1]

(b) Using the substitution $u = \sqrt{x}$, find the exact area of the shaded region in the first quadrant bounded by this part of the curve and the x-axis. [7]

- w22_qp_33

Q5

(a) On a sketch of an Argand diagram, shade the region whose points represent complex numbers z satisfying the inequalities $|z + 2| \leqslant 2$ and $\text{Im}\, z \geqslant 1$. [4]

(b) Find the greatest value of $\arg z$ for points in the shaded region. [2]

Q6

Solve the quadratic equation $(1 - 3\text{i})z^2 - (2 + \text{i})z + \text{i} = 0$, giving your answers in the form $x + \text{i}y$, where x and y are real. [6]

Q9

With respect to the origin O, the position vectors of the points A, B and C are given by

$$\overrightarrow{OA} = \begin{pmatrix} 0 \\ 5 \\ 2 \end{pmatrix}, \quad \overrightarrow{OB} = \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} \quad \text{and} \quad \overrightarrow{OC} = \begin{pmatrix} 4 \\ -3 \\ -2 \end{pmatrix}.$$

The midpoint of AC is M and the point N lies on BC, between B and C, and is such that $BN = 2NC$.

(a) Find the position vectors of M and N. [3]

(b) Find a vector equation for the line through M and N. [2]

(c) Find the position vector of the point Q where the line through M and N intersects the line through A and B. [4]

Q10

A gardener is filling an ornamental pool with water, using a hose that delivers 30 litres of water per minute. Initially the pool is empty. At time t minutes after filling begins the volume of water in the pool is V litres. The pool has a small leak and loses water at a rate of $0.01V$ litres per minute.

The differential equation satisfied by V and t is of the form $\dfrac{dV}{dt} = a - bV$.

(a) Write down the values of the constants a and b. [1]

(b) Solve the differential equation and find the value of t when $V = 1000$. [6]

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Q2

(a) On an Argand diagram, shade the region whose points represent complex numbers z satisfying the inequalities $-\frac{1}{3}\pi \leqslant \arg(z - 1 - 2i) \leqslant \frac{1}{3}\pi$ and $\operatorname{Re} z \leqslant 3$. [3]

(b) Calculate the least value of $\arg z$ for points in the region from **(a)**. Give your answer in radians correct to 3 decimal places. [2]

Q3

The polynomial $2x^4 + ax^3 + bx - 1$, where a and b are constants, is denoted by $p(x)$. When $p(x)$ is divided by $x^2 - x + 1$ the remainder is $3x + 2$.

Find the values of a and b. [5]

Q4

Solve the equation

$$\frac{5z}{1+2i} - zz^* + 30 + 10i = 0,$$

giving your answers in the form $x + iy$, where x and y are real. [5]

Q9

The variables x and y satisfy the differential equation

$$\frac{dy}{dx} = e^{3y}\sin^2 2x.$$

It is given that $y = 0$ when $x = 0$.

Solve the differential equation and find the value of y when $x = \frac{1}{2}$. [7]

Q11

Let $f(x) = \dfrac{5x^2 + x + 11}{(4 + x^2)(1 + x)}$.

(a) Express $f(x)$ in partial fractions. [5]

(b) Hence show that $\int_0^2 f(x)\,dx = \ln 54 - \frac{1}{8}\pi$. [5]

- s23_qp_31

Q7

The variables x and y satisfy the differential equation

$$\cos 2x \frac{dy}{dx} = \frac{4\tan 2x}{\sin^2 3y},$$

where $0 \leqslant x < \frac{1}{4}\pi$. It is given that $y = 0$ when $x = \frac{1}{6}\pi$.

Solve the differential equation to obtain the value of x when $y = \frac{1}{6}\pi$. Give your answer correct to 3 decimal places. [8]

Q9

The constant a is such that $\int_0^a xe^{-2x}\,dx = \frac{1}{8}$.

(a) Show that $a = \frac{1}{2}\ln(4a + 2)$. [5]

(b) Verify by calculation that a lies between 0.5 and 1. [2]

(c) Use an iterative formula based on the equation in **(a)** to determine a correct to 2 decimal places. Give the result of each iteration to 4 decimal places. [3]

Q10

The polynomial $x^3 + 5x^2 + 31x + 75$ is denoted by $p(x)$.

(a) Show that $(x + 3)$ is a factor of $p(x)$. [2]

(b) Show that $z = -1 + 2\sqrt{6}i$ is a root of $p(z) = 0$. [3]

(c) Hence find the complex numbers z which are roots of $p(z^2) = 0$. [7]

- s23_qp_32

Q4

Solve the equation $2\cos x - \cos\frac{1}{2}x = 1$ for $0 \leqslant x \leqslant 2\pi$. [5]

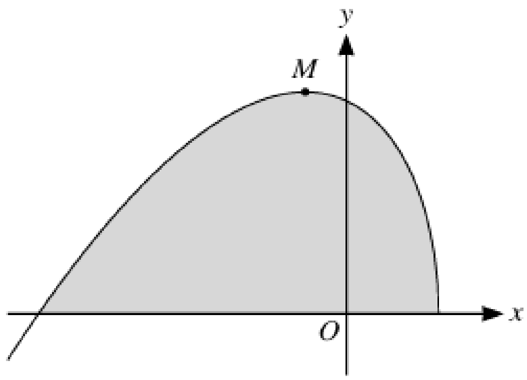
Q9

Let $f(x) = \dfrac{2x^2 + 17x - 17}{(1 + 2x)(2 - x)^2}$.

(a) Express $f(x)$ in partial fractions. [5]

(b) Hence show that $\int_0^1 f(x)\,dx = \frac{5}{2} - \ln 72$. [5]

Q10



The diagram shows the curve $y = (x+5)\sqrt{3-2x}$ and its maximum point M.

(a) Find the exact coordinates of M. [5]

(b) Using the substitution $u = 3 - 2x$, find by integration the area of the shaded region bounded by the curve and the x-axis. Give your answer in the form $a\sqrt{13}$, where a is a rational number. [5]

- s23_qp_33

Q8

The variables x and y satisfy the differential equation

$$\frac{dy}{dx} = \frac{y^2+4}{x(y+4)}$$

for $x > 0$. It is given that $x = 4$ when $y = 2\sqrt{3}$.

Solve the differential equation to obtain the value of x when $y = 2$. [8]

Q9

The lines l and m have equations

$$l: \quad \mathbf{r} = a\mathbf{i} + 3\mathbf{j} + b\mathbf{k} + \lambda(c\mathbf{i} - 2\mathbf{j} + 4\mathbf{k}),$$
$$m: \quad \mathbf{r} = \mathbf{i} + 2\mathbf{j} + 3\mathbf{k} + \mu(2\mathbf{i} - 3\mathbf{j} + \mathbf{k}).$$

Relative to the origin O, the position vector of the point P is $4\mathbf{i} + 7\mathbf{j} - 2\mathbf{k}$.

(a) Given that l is perpendicular to m and that P lies on l, find the values of the constants a, b and c. [4]

(b) The perpendicular from P meets line m at Q. The point R lies on PQ extended, with $PQ : QR = 2 : 3$.

Find the position vector of R. [6]

Q11

The complex number z is defined by $z = \dfrac{5a - 2i}{3 + ai}$, where a is an integer. It is given that $\arg z = -\frac{1}{4}\pi$.

(a) Find the value of a and hence express z in the form $x + iy$, where x and y are real. [6]

(b) Express z^3 in the form $re^{i\theta}$, where $r > 0$ and $-\pi < \theta \leqslant \pi$. Give the simplified exact values of r and θ. [3]

A Note from Mojza

These notes for A Level Mathematics (9709) have been prepared by Team Mojza, covering the content for A Level 2022-24 syllabus. The content of these notes has been prepared with utmost care. We apologise for any issues overlooked; factual, grammatical or otherwise. We hope that you benefit from these and find them useful towards achieving your goals for your Cambridge examinations.

If you find any issues within these notes or have any feedback, please contact us at support@mojza.org.

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