MOJZA

AS Level

Paper 1 - Solutions

9709/01

BY TEAM MOJZA

CONTENTS

QUADRATICS

Completing the square

9709_w21_13 -q3

Express $5y^2 - 30y + 50$ in the form $5(y + a)^2 + b$, where a and b are constants.

$5y^2 - 30y + 50$

$5(y^2 - 6y + 6/2^2) + (5 \times -(3)^2 + 50)$

$5(y^2 - 6y + 3^2) + (-45 + 50)$

$5(y - 3)^2 + 5$

$a = -3$

$b = 5$

9709_s18_13 -q1

Express $3x^2 - 12x + 7$ in the form $a(x + b)^2 + c$, where a, b and c are constants.

$3x^2 - 12x + 7$

$3(x^2 - 4x + 4/2^2) + (3 \times -(2)^2 + 7)$

$3(x^2 - 4x + 2^2) + (-12 + 7)$

$3(x - 2)^2 - 5$

$a = 3 \quad c = -5$

$b = -2$

Discriminant:

9709_S14_11

Find the set of values of k for which the line $y = 2x - k$ meets the curve $y = x^2 + kx - 2$ at two distinct points. [5]

① Equate both equations

$$2x - k = x^2 + kx - 2$$

② Form a quadratic

$$x^2 - 2x + kx - 2 + k = 0$$

$$x^2 + (k-2)x - 2 + k = 0$$

③ Use discriminant "$b^2 - 4ac$"

$$(k-2)^2 - 4(1)(-2+k) > 0$$

$$k^2 - 4k + 4 + 8 - 4k > 0$$

$$k^2 - 8k + 12 > 0$$

$$(k-6)(k-2) > 0$$

$$k > 6 \qquad k > 2$$

flip inequality of smaller number

$$k > 6 \quad \text{and} \quad k < 2.$$

Q4) 9709_M17_12

Find the set of values of k for which the equation $2x^2 + 3kx + k = 0$ has distinct real roots.

$2x^2 + 3kx + k = 0$

$b^2 - 4ac \geq 0$

$(3k)^2 - 4(2)(k) \geq 0$

$9k^2 - 8k \geq 0$

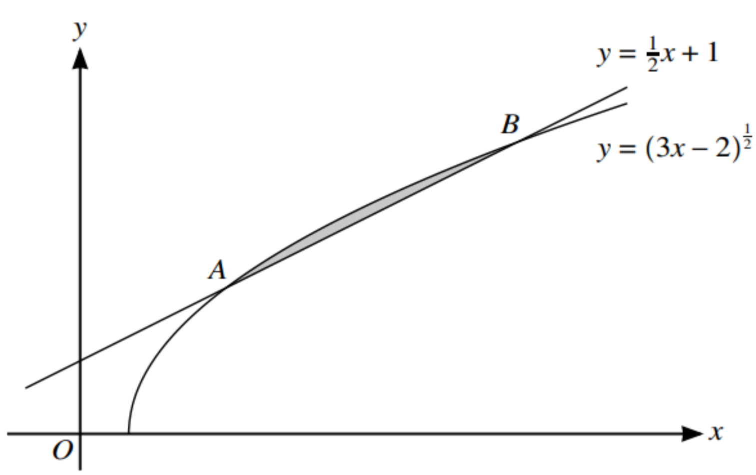
$k(9k - 8) > 0$

flip! $k > 0$ $\quad k \geq \dfrac{8}{9}$

$k < 0$

Solutions To A Graph:

Q5) 9709_s22_qp_11



The diagram shows the curve with equation $y = (3x - 2)^{\frac{1}{2}}$ and the line $y = \frac{1}{2}x + 1$. The curve and the line intersect at points A and B.

(a) Find the coordinates of A and B. [4]

• Equate both equations

$(3x-2)^{1/2} = 0.5x+1$

$(3x-2) = (0.5x+1)^2$

$(3x-2) = (0.25x^2 + x + 1)$

$0.25x^2 - 3x + x + 2 + 1$

$\frac{1}{4}x^2 - 2x + 3 = 0$

$x = 6$ middle term $x = 2$

$y = \frac{1}{2}(6) + 1 = 4$ $\quad$ $y = \frac{1}{2}(2) + 1 = 2$

$(6, 4)$ $\quad$ $(2, 2)$

Q6) 9709_w21_qp_13

The line $y = 2x + 5$ intersects the circle with equation $x^2 + y^2 = 20$ at A and B.

(a) Find the coordinates of A and B in surd form and hence find the exact length of the chord AB. [7]

LINE AND CIRCLE.

① EQUATE

$y = 2x + 5$ $\quad$ $y = \sqrt{20 - x^2}$

$2x + 5 = (20 - x^2)^{\frac{1}{2}}$

$(2x+5)^2 = 20 - x^2$

$4x^2 + 20x + 25 = 20 - x^2$

$5x^2 + 20x + 5 = 0$

$x = -2 + \sqrt{3}$ $\quad$ $x = -2 - \sqrt{3}$

$y = 2(-2+\sqrt{3}) + 5 = 1 + 2\sqrt{3}$ $\quad$ $y = 2(-2-\sqrt{3}) + 5 = 1 - 2\sqrt{3}$

A $(-2+\sqrt{3}, 1+2\sqrt{3})$ $\quad$ B $(-2-\sqrt{3}, 1-2\sqrt{3})$

$\overline{AB} = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2} = \sqrt{60}$

COORDINATE GEOMETRY

Equation Of Perpendicular Bisector Of A Line

Q1) 9709_S19_QP_12

Two points A and B have coordinates $(1, 3)$ and $(9, -1)$ respectively. The perpendicular bisector of AB intersects the y-axis at the point C. Find the coordinates of C. [5]

8) $A(1, 3)$ $B(9, -1)$

$$m = \frac{-1-3}{9-1} = -\frac{1}{2}$$

① Make equation of perpendicular bisector using gradient and midpoint.

gradient $\rightarrow 2$

midpoint $\rightarrow \left(\frac{1+9}{2}, \frac{3+(-1)}{2}\right) = (5, 1)$

$$Y - y_1 = m(X - x_1)$$
$$Y - 1 = 2(X - 5)$$
$$Y = 2x - 9$$

② Put "$x = 0$" to find y-axis intersection.

$$Y = 2x - 9$$
$$Y = 2(0) - 9$$
$$Y = -9$$

$$C = (0, -9)$$

Using Expanded Formula Of Circle

Q2) 9709_s22_qp_11

The equation of a circle is $x^2 + y^2 + 6x - 2y - 26 = 0$.

(a) Find the coordinates of the centre of the circle and the radius. Hence find the coordinates of the lowest point on the circle. [4]

$x^2 + y^2 + 6x - 2y - 26 = 0$

(i) arrange $\quad x^2 + 6x + y^2 - 2y = 26$

(ii) form identity. $\quad x^2 + 6x + (6/2)^2 + y^2 - 2y + (2/2)^2 = 26 + \left(\frac{6}{2}\right)^2 + \left(\frac{2}{2}\right)^2$

(iii) solve $\quad x^2 + 6x + 3^2) + y^2 - 2y + 1 = 26 + 9 + 1$

(iv) find centre & radius $\quad (x+3)^2 (y-1)^2 = 36$

centre $\longrightarrow (-3, 1)$

radius $\longrightarrow \sqrt{36} = 6$

Circle Properties

Q3) 9709_s20_qp_11

The coordinates of the points A and B are $(-1, -2)$ and $(7, 4)$ respectively.

(a) Find the equation of the circle, C, for which AB is a diameter. [4]

A $(-1, -2)$ $\quad$ B $(7, 4)$

1. Centre is the midpoint of these 2 points.

m.p $\longrightarrow \left(\frac{7 + (-1)}{2} , \frac{4 + (-2)}{2} \right)$

$= 3 , 1$

2. radius is distance from midpoint to point on e[...]

$= \sqrt{(x_2 \mp x_1)^2 + (y_2 - y_1)^2}$

$= \sqrt{(7-3)^2 + (1-4)^2}$

$= \sqrt{16 + 9}$

$= \sqrt{25}$

$= 5$

3. substita values in equation form:

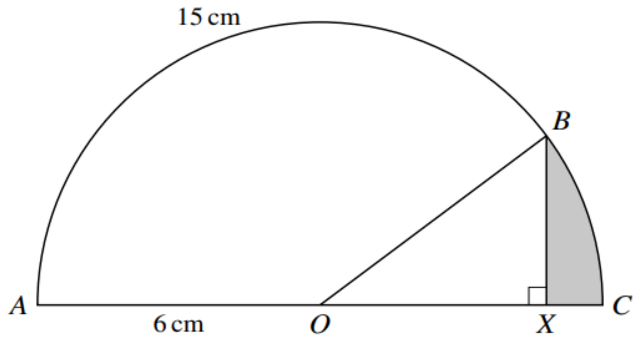
$(x - x_1)^2 + (y - y_1)^2 = r^2$

$(x-3)^2 + (y-1)^2 = 25$ $\boxed{\text{ANS}}$

CIRCULAR MEASURE

Finding Arc Length of Shaded Region

Q) 9709_s20_qp_11



In the diagram, ABC is a semicircle with diameter AC, centre O and radius 6 cm. The length of the arc AB is 15 cm. The point X lies on AC and BX is perpendicular to AX.

Find the perimeter of the shaded region BXC. [6]

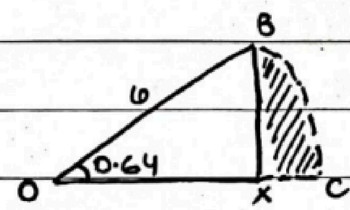
i) Find BOC using $S = r\theta$

$S = r\theta$

$15 = 6(\theta)$

$\theta = 2.5$

$\pi - \theta = 0.64$ - BOC



ii) BX $\longrightarrow$ $6\sin(0.64) = 3.58$

OX $\longrightarrow$ $6\cos(0.64) = 4.80$

XC $\longrightarrow$ $6 - 4.80 = 1.19$

BC $\longrightarrow$ $r\theta = 6(0.64) = 3.85$

iii) Perimeter : $BX + XC + BC$

$3.58 + 1.19 + 3.85$

8.634

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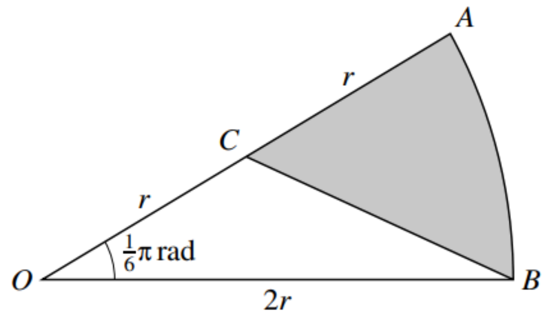
Q) 9709_s20_qp_12



In the diagram, OAB is a sector of a circle with centre O and radius $2r$, and angle $AOB = \frac{1}{6}\pi$ radians. The point C is the midpoint of OA.

(a) Show that the exact length of BC is $r\sqrt{5 - 2\sqrt{3}}$. [2]

1. Use cosine rule:

$$c^2 = a^2 + b^2 - 2ab(\cos\theta)$$

$$BC^2 = r^2 + (2r)^2 - 2(r)(2r)(\cos \pi/6)$$

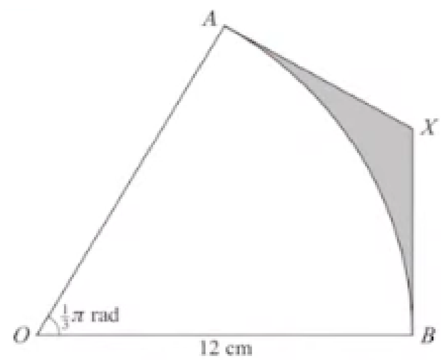
$$BC^2 = 5r^2 - 2\sqrt{3}r^2$$

$$BC^2 = r^2(5 - 2\sqrt{3})$$

$$BC = \sqrt{r^2(5-2\sqrt{3})}$$

$$BC = r\sqrt{5-2\sqrt{3}}$$

FINDING AREA OF SHADED REGION :



In the diagram, OAB is a sector of a circle with centre O and radius 12 cm. The lines AX and BX are tangents to the circle at A and B respectively. Angle $AOB = \frac{1}{3}\pi$ radians.

(i) Find the exact length of AX, giving your answer in terms of $\sqrt{3}$. [2]

(ii) Find the area of the shaded region, giving your answer in terms of π and $\sqrt{3}$. [3]

Solution:

(i) AX $\longrightarrow$ $\tan\theta = \frac{\text{Opp}}{\text{Adj}}$

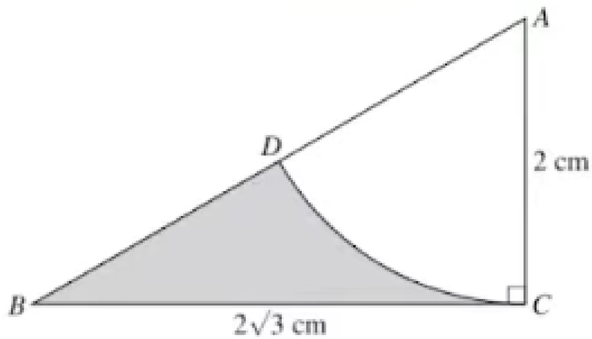
$$\tan\left(\frac{\pi}{6}\right) = \frac{AX}{12}$$

$$4\sqrt{3} = AX$$

(ii) (2 × area of Δ) − (area of sector)

$$\left(2 \times \frac{1}{2} \times 12 \times 4\sqrt{3}\right) - \left(\frac{1}{2} \times 12^2 \times \frac{\pi}{3}\right)$$

$$48\sqrt{3} - 24\pi$$



In the diagram, D lies on the side AB of triangle ABC and CD is an arc of a circle with centre A and radius 2 cm. The line BC is of length $2\sqrt{3}$ cm and is perpendicular to AC. Find the area of the shaded region BDC, giving your answer in terms of π and $\sqrt{3}$. [4]

i. Angle BAC $\rightarrow$ $\tan\theta = \frac{2\sqrt{3}}{2}$

$\theta = \frac{\pi}{3}$

ii. Area = Δ − D

$\left(\frac{1}{2} \times 2\sqrt{3} \times 2\right) - \left(\frac{1}{2} \times 2^2 \times \frac{\pi}{3}\right)$

$2\sqrt{3} - \frac{2}{3}\pi$

SERIES AND PROGRESSION

Formula Based

9709_s22_qp_11

The thirteenth term of an arithmetic progression is 12 and the sum of the first 30 terms is −15.

Find the sum of the first 50 terms of the progression. [5]

SOLUTION:

1. $T = a + (n-1)d$

$a + 12d = 12$

$a = 12 - 12d$

2. $S_{30} = -15$.

$\frac{30}{2}\left(2a + 29d\right) = -15$

$15\left(2(12-12d) + 29d\right) = -15$

$24 - 24d + 29d = -1$

$5d = -25$

$d = -5$

$a = 12 - 12(-5) = 72$

3. S_{50}:

$\frac{50}{2}\left(2(72) + 49(-5)\right)$

$\boxed{-2525 \quad \text{ANS}}$

9709_w22_qp_13

The first term of a geometric progression is 216 and the fourth term is 64.

(a) Find the sum to infinity of the progression. [3]

SOLUTION :

(i) find r $\rightarrow$ Term 4 = ar^3

$64 = (216)r^3$

$\sqrt[3]{\frac{8}{27}} = r = \frac{2}{3}$

(ii) Use formula $\rightarrow$ $S_\infty = \frac{a}{1-r}$

$= \frac{216}{1-2/3} = 648$

Scenario Based

9709_s20_qp_11

Each year the selling price of a diamond necklace increases by 5% of the price the year before. The selling price of the necklace in the year 2000 was \$36 000.

(a) Write down an expression for the selling price of the necklace n years later and hence find the selling price in 2008. [3]

(b) The company that makes the necklace only sells one each year. Find the total amount of money obtained in the ten-year period starting in the year 2000. [2]

Solution:

(i) · Increase by 5% = 1.05.

$a = 36000$

$T = ar^n$

$T = 36000(1.05)^n$

$T_8 = 36000(1.05)^8$

$= \$53200$

(ii) $S_{10} = \frac{36000(1.05^{10}-1)}{1.05-1}$

$= \$453000$

DIFFERENTIATION

Equation Of Tangent

9709_s21_qp_11

The equation of a curve is $y = 2\sqrt{3x+4} - x$.

(a) Find the equation of the normal to the curve at the point (4, 4), giving your answer in the form $y = mx + c$. [5]

$y = 2\sqrt{3x+4} - x$

(i) First find gradient by diffrentiating.

$y = 2(3x+4)^{1/2} - x$

$= 2\left(\frac{1}{2}\right)(3x+4)^{-1/2} \times 3 - 1$

$\frac{dy}{dx} = \frac{3}{\sqrt{3x+4}} - 1$

(ii). Put (4, 4) to find tangent m then normal.

$m = \frac{3}{\sqrt{3(4)+4}} - 1 = -\frac{1}{4}$

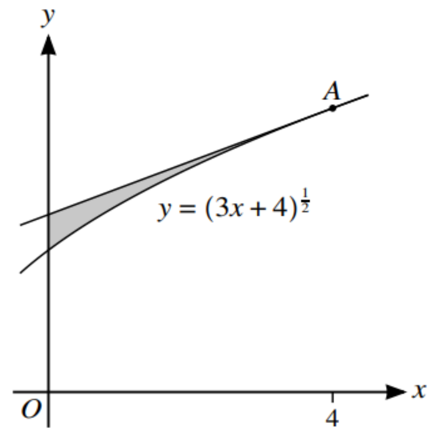
normal = 4.

(iii) make equation using $y - y_1 = m(x - x_1)$

$y - 4 = 4(x - 4)$

$y = 4x - 12$

9709_s19_qp_13



The diagram shows part of the curve with equation $y = (3x+4)^{\frac{1}{2}}$ and the tangent to the curve at the point A. The x-coordinate of A is 4.

(i) Find the equation of the tangent to the curve at A. [5]

Solution →

(i) find (x, y)

$x = 4$

$y = (3(4)+4)^{\frac{1}{2}} = 4.$

$(4, 4)$

(ii) diffrentiate :

$y = (3x+4)^{\frac{1}{2}}$

$= \frac{1}{2}(3x+4)^{-\frac{1}{2}} \times 3$

$= \frac{3}{2}(3x+4)^{-\frac{1}{2}}$

Put $x=4 \rightarrow \frac{3}{2}(3(4)+4)^{-\frac{1}{2}} = \frac{3}{8}.$

EQ = $Y - 4 = \frac{3}{8}(x-4)$

$Y = 3/8x + 5/2$

dy/dx and Stationary Points

9709_s20_qp_11

The equation of a curve is $y = (3 - 2x)^3 + 24x$.

(a) Find expressions for $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$. [4]

(b) Find the coordinates of each of the stationary points on the curve. [3]

i) $y = (3-2x)^3 + 24x$

$\frac{dy}{dx} = 3(3-2x)^2(-2) + 24$

$= -6(3-2x)^2 + 24$

$\frac{d^2y}{dx^2} = -12(3-2x)(-2)$

$= 24(3-2x)$

(ii) stationary point. $dy/dx = 0$.

$-6(3-2x)^2 + 24 = 0$

$(3-2x)^2 = 4$

$3 - 2x = \pm 2$

$3-2x = -2$ | $3-2x = 2$

$x = 2.5$, $y = \cancel{64}\,52$ | $x = 0.5$, $y = 20$

9709_s20_qp_12

The equation of a curve is $y = 54x - (2x - 7)^3$.

(a) Find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$. [4]

(b) Find the coordinates of each of the stationary points on the curve. [3]

(c) Determine the nature of each of the stationary points. [2]

(i) $y = 54x - (2x-7)^3$

$\rightarrow 54 - 3(2x-7)^2(2)$

$\frac{dy}{dx} = 54 - 6(2x-7)^2$

$\rightarrow -12(2x-7)(2)$

$\frac{d^2y}{dx^2} = -24(2x-7)$

(ii) $54x - 6(2x-7)^2 = 0$

$(2x-7)^2 = 9$

$2x-7 = 3$ $\quad 2x-7 = -3$

$x = 5$ $y = 243$ $\quad x = 2$ $y = 135$

(iii) nature:

$x = 5$ $\quad -24(2(5)-7) = -72$ max!

$x = 2$ $\quad -24(2(2)-7) = 72$ min!

Rate Change

9709_w22_qp_13

A large industrial water tank is such that, when the depth of the water in the tank is x metres, the volume $V\,\text{m}^3$ of water in the tank is given by $V = 243 - \frac{1}{3}(9-x)^3$. Water is being pumped into the tank at a constant rate of $3.6\,\text{m}^3$ per hour.

Find the rate of increase of the depth of the water when the depth is 4 m, giving your answer in cm per minute. [5]

$$V = 243 - 1/3(9-x)^3$$

$$\frac{dV}{dx} = (9-x)^2$$

$$\frac{dV}{dt} = 3.6$$

$$\frac{dx}{dt} = \frac{1}{(9-x)^2} \times 3.6$$

$$= \frac{1}{(9-4)^2} \times 3.6 = 0.144$$

$$0.144\,\text{m}^3/\text{h} \rightarrow \text{cm}^3/\text{min}$$

$$0.144 \times \frac{100}{60} = \underline{\underline{0.24}}$$

9709_w21_qp_12.

The volume V m^3 of a large circular mound of iron ore of radius r m is modelled by the equation $V = \frac{3}{2}(r - \frac{1}{2})^3 - 1$ for $r \geqslant 2$. Iron ore is added to the mound at a constant rate of 1.5 m^3 per second.

(a) Find the rate at which the radius of the mound is increasing at the instant when the radius is 5.5 m. [3]

$$V = 3/2\,(r - 1/2)^3 - 1$$

$$\frac{dV}{dr} \rightarrow 9/2\,(r - 1/2)^2$$

$$r = 5.5$$

$$= 9/2\,(5.5 - 1/2)^2$$

$$\frac{dV}{dr} = 112.5$$

$$\frac{dr}{dt} = \frac{dV}{dt} \times \frac{dr}{dV}$$

$$\frac{dr}{dt} = 1.5 \times \frac{1}{112.5}$$

$$= \frac{1}{75}\ \text{m/s.}$$

INTEGRATION

Equation Of Curve

9709_s22_qp_11

The equation of a curve is such that $\frac{d^2y}{dx^2} = 6x^2 - \frac{4}{x^3}$. The curve has a stationary point at $\left(-1, \frac{9}{2}\right)$.

Find the equation of the curve.

$\frac{d^2y}{dx^2} \rightarrow \frac{dy}{dx}$

$6x^2 - 4x^{-3}$

$\frac{6x^3}{3} - \frac{4x^{-2}}{-2}$

$\frac{dy}{dx} = 2x^3 + 2x^{-2} + c$

$\left(0 = {}^{dy}/_{dx},\ x = -1\right)$

$0 = 2(-1)^3 + 2(-1)^2 + c$

$c = 0$.

$\frac{dy}{dx} \longrightarrow y$

$2x^3 + 2x^{-2}$

$\frac{2x^4}{4} + \frac{2x^{-1}}{-1}$

$y = \frac{1}{2}x^4 - 2x^{-1} + c$

$\left(-1, {}^{9}/_{2}\right)$

${}^{9}/_{2} = {}^{1}/_{2}(-1)^4 - 2(-1)^{-1} + c$

$c = 2$.

equation: $y = \frac{1}{2}x^4 - \frac{2}{x} + 2$

9709_s21_qp_11

The equation of a curve is such that $\frac{dy}{dx} = \frac{3}{x^4} + 32x^3$. It is given that the curve passes through the point $\left(\frac{1}{2}, 4\right)$.

Find the equation of the curve. [4]

${}^{dy}/_{dx} = 3x^{-4} + 32x^3$

$\frac{dy}{dx} = y \rightarrow \frac{3x^{-3}}{-3} + \frac{32x^4}{4}$

$y = \frac{-1}{x^3} + 8x^4 + c$

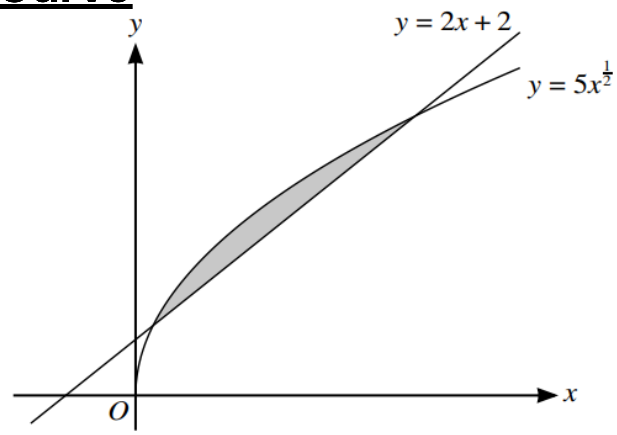
$(0.5 ; 4)$

$4 = \frac{-1}{(0.5)^3} + 8(0.5)^4 + c$

$c = 11.5$

eq $\rightarrow y = \frac{-1}{x^3} + 8x^4 + \frac{23}{2}$

Area Under Curve



The diagram shows the curve with equation $y = 5x^{\frac{1}{2}}$ and the line with equation $y = 2x + 2$.

Find the exact area of the shaded region which is bounded by the line and the curve. [5]

AREA:

① first find "x-coordinate" limit by point of intersection

$y = 2x + 2 \qquad y = 5x^{1/2}$

$2x + 2 = 5x^{1/2}$

$2x - 5x^{1/2} + 2 = 0$

(solve quadratically)

$x = 0{\cdot}25 \qquad x = 4.$

③ Integrate using limits (upper curve) − (lower curve)

$y = \int_{0.25}^{4} 5x^{1/2} \quad - \quad y = \int 2x + 2$

$= \left| \frac{5x^{3/2}}{3/2} \right|_{0.25}^{4} - \left| \frac{2x^2}{2} + 2x \right|_{0.25}^{4}$

$\left(\frac{10}{3}(4)^{3/2}\right) - \left(\frac{10}{3}(0.25)^{3/2}\right) - \left((4)^2 + 2(4) - (0.25)^2 + 2(0.25)\right)$

$\frac{105}{4} - \frac{375}{16}$

$= \frac{45}{16}$

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Q2)

Find the area of the region enclosed by the curve $y = 2\sqrt{x}$, the x-axis and the lines $x = 1$ and $x = 4$. [4]

(1) integrate

$y = \int 2x^{1/2}$

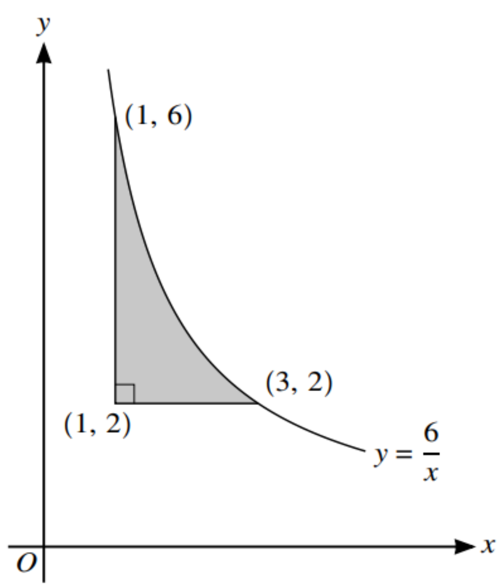
$\frac{2x^{3/2}}{3/2}$

(2) Limit: $\left. \frac{4}{3} x^{3/2} \right|_1^4$

$\frac{4}{3}(4)^{3/2} - \frac{4}{3}(1)^{3/2} = \frac{28}{3}$

Volume Under Curve

9709_s20_qp_12



The diagram shows part of the curve $y = \frac{6}{x}$. The points (1, 6) and (3, 2) lie on the curve. The shaded region is bounded by the curve and the lines $y = 2$ and $x = 1$.

(a) Find the volume generated when the shaded region is rotated through 360° about the ***y*-axis**. [5]

"about y-axis" → make x subject.

$x = \frac{6}{y}$

UPPER CURVE

$x^2 = 36y^{-2}$

integrate: $\int_2^6 36y^{-2}$

$\left| -36y^{-1} \right|_2^6$

$\left| \left(-36(6)^{-1} \right) - \left(-36(2)^{-1} \right) \right|$

$\left| (-6) - (-18) \right|$

$= 12 \times \pi \quad = 12\pi$

CYLINDER

$= \pi r^2 h$

$= \pi (1)^2 4$

$= 4\pi$

FINAL $= 12\pi - 4\pi$

$= 8\pi$

TRIGONOMETRY

Prove The Identity

9709_s20_qp_11

(a) Prove the identity $\frac{1 + \sin\theta}{\cos\theta} + \frac{\cos\theta}{1 + \sin\theta} \equiv \frac{2}{\cos\theta}$. [3]

Solution :

(1) LCM: $\frac{(1+\sin\theta)(1+\sin\theta) + (\cos\theta)(\cos\theta)}{\cos\theta(1+\sin\theta)}$

$\Rightarrow \frac{1+\sin^2\theta+\sin\theta+\sin\theta+\cos^2\theta}{\cos\theta(1+\sin\theta)}$

$\Rightarrow \frac{1+1+2\sin\theta}{\cos\theta(1+\sin\theta)}$

$\Rightarrow \frac{2\cancel{(1+\sin\theta)}}{\cos\theta\cancel{(1+\sin\theta)}}$

$\Rightarrow \frac{2}{\cos\theta}$

9709_w20_qp_11

(a) Show that $\frac{\sin\theta}{1 - \sin\theta} - \frac{\sin\theta}{1 + \sin\theta} \equiv 2\tan^2\theta$. [3]

(1) LCM:

$\frac{\sin\theta(1+\sin\theta) - \sin\theta(1-\sin\theta)}{1-\sin^2\theta}$

$\frac{\sin^2\theta + \cancel{\sin\theta} - \cancel{\sin\theta} + \sin^2\theta}{1-\sin^2\theta}$

$\frac{2\sin^2\theta}{\cos^2\theta} = 2\tan^2\theta$

Find The Angle

9709_w22_qp_13

Solve the equation $8\sin^2\theta + 6\cos\theta + 1 = 0$ for $0° < \theta < 180°$. [3]

SOLUTION

$8(1-\cos^2\theta) + 6\cos\theta + 1 = 0$

$8 - 8\cos^2\theta + 6\cos\theta + 1 = 0$

$8c^2 - 6c - 9 = 0$

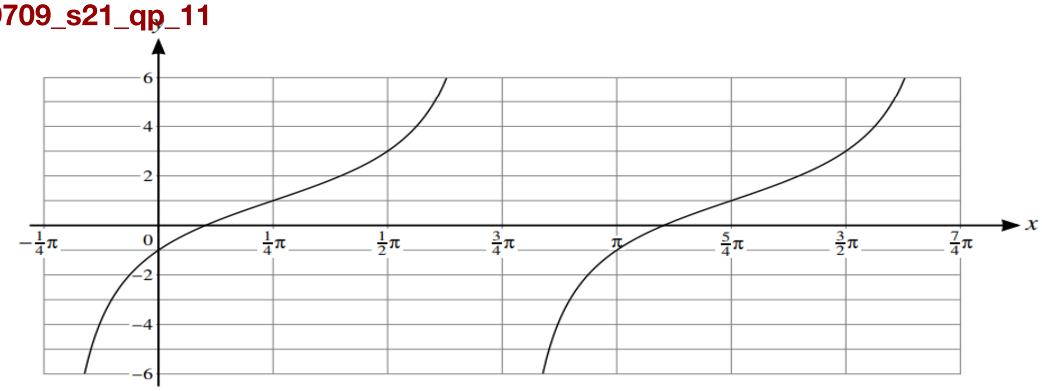
$\cos\theta = 3/2$ $\quad\quad$ $\cos\theta = -3/4$

$\theta =$ not pos. $\quad\quad$ $\theta = 41.41$

ans $= 138.6°$ (neg quadrant)

Graph

9709_s21_qp_11



The diagram shows part of the graph of $y = a\tan(x - b) + c$.

Given that $0 < b < \pi$, state the values of the constants a, b and c. [3]

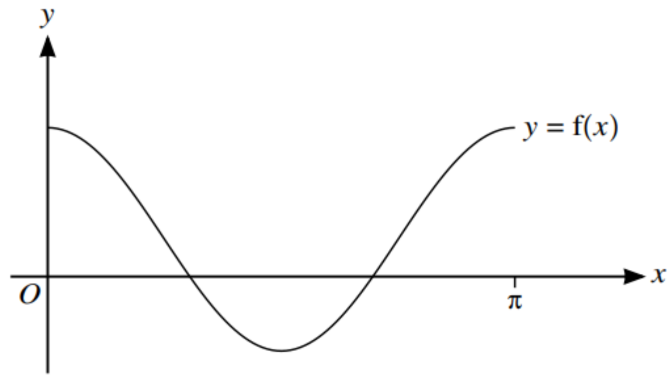
SOLUTION:

$a =$ amplitude $= 2$

$b =$ transformed right by $\pi/4$.

$c = 1$ (constant line.

9709_s20_qp_11



The diagram shows the graph of $y = \mathrm{f}(x)$, where $\mathrm{f}(x) = \frac{3}{2}\cos 2x + \frac{1}{2}$ for $0 \leqslant x \leqslant \pi$.

(a) State the range of f. [2]

To find range of f:

sub / add amplitude to constant line.

$$\underbrace{\frac{3}{2}\cos 2x}_{\text{amplitude}} + \underbrace{\frac{1}{2}}_{\text{constant line}}$$

- $\frac{1}{2} + \frac{3}{2} = 2$
- $\frac{1}{2} - \frac{3}{2} = -1$

$= -1 \leq f(x) \leq 2$

FUNCTIONS:

With Differentiation

9709_w21_qp_13

(b) The function f is defined by $f(x) = x^5 - 10x^3 + 50x$ for $x \in \mathbb{R}$.

Determine whether f is an increasing function, a decreasing function or neither. [3]

Solution: diffrentiate first.

$f(x) = x^5 - 10x^3 + 50x$

$\frac{dy}{dx} = 5x^4 - 30x^2 + 50$

factorise now:-

$5x^2(x^2 - 6 + 3^2) - (3)^2 \times 5 + 50$

$5x^2(x^2 - 3)^2 + 5$

∴ Hence, as, for any value of x

$(x^2-3)^2$ will be > 0

$5(x^2-3)^2$ will be > 0

$5(x^2-3)^2 + 5$ will be > 0

as always positive | It is increasing function

Composite Functions

9709_w21_qp_12.

The function f is defined as follows:

$$f(x) = \frac{x+3}{x-1} \text{ for } x > 1.$$

(a) Find the value of ff(5). [2]

$f(x) = \dfrac{x+3}{x-1}$ input 5 input $f(5)$ in x

• $f(5) = \dfrac{5+3}{5-1} = 2$ • $ff(5) = \dfrac{2+3}{2-1} = \underline{\underline{5}}$

Transformations

9709_w21_qp_12.

The graph of $y = f(x)$ is transformed to the graph of $y = f(2x) - 3$.

(a) Describe fully the two single transformations that have been combined to give the resulting transformation. [3]

$y = \underbrace{f(2x)}_{1}\underbrace{-3}_{2}$

1: strech by factor of $\frac{1}{2}$ in x-direction

2: translate by $\begin{pmatrix} 0 \\ -3 \end{pmatrix}$

9709_w21_qp_13

The graph of $y = f(x)$ is transformed to the graph of $y = 3 - f(x)$.

Describe fully, in the correct order, the two transformations that have been combined. [4]

$y = \underbrace{3}_{2}\underbrace{-f(x)}_{1}$

1: reflection in x-axis

2: translate by $\begin{bmatrix} 0 \\ 3 \end{bmatrix}$

BINOMIAL THEOREM

Expand First few Terms

Q1) 9709_w21_qp_13

(a) Find the first three terms, in ascending powers of x, in the expansion of $(1 + ax)^6$.

$(1 + ax)^6$

${}^6C_0\, ax^0 1^6 + {}^6C_1\, ax^1 . 1^5 + {}^6C_2\, ax^2 1^4$

$1 + 6ax + 15a^2x^2$.

Q2) 9709_w22_qp_13

(a) Find the first three terms in ascending powers of x of the expansion of $(1 + 2x)^5$.

$(1+2x)^5$ $\qquad {}^nC_r(b^r)(a^{n-r})$

${}^5C_0\, 2x^0 1^5 + {}^5C_1\, 2x^1 1^4 + {}^5C_2\, 2x^2 1^3 \ldots$

$1 + 10x + 40x^2$

FIND COEFFICIENT OF SPECIFIC TERMS

9709_s21_qp_11

(b) Hence find the coefficient of x^2 in the expansion of $(4+x)^2(3-2x)^5$. [3]

coff of x^2 :

$(4+x)^2(3-2x)^5$

$(16+8x+x^2)(^5C_0 2x^0 3^5 - {^5C_1} 2x^1 3^4 + {^5C_2} 2x^2 3^3 \ldots)$

$(16+8x+x^2)(243 - 810x + 1080x^2 \ldots)$

$243x^2 - 6480x^2 + 17280x^2$

$11043x^2$

$= 11043$

9709_w21_qp_12.

(a) It is given that in the expansion of $(4+2x)(2-ax)^5$, the coefficient of x^2 is -15.

Find the possible values of a. [4]

coff of $x^2 = -15$

$(4+2x)(2-ax)^5$

$(4+2x)(^5C_0 ax^0 2^5 - {^5C_1} ax^1 2^4 + {^5C_2} ax^2 2^3 \ldots)$

$(4+2x)(32 - 80ax + 80a^2x^2 \ldots)$

$-160ax^2 + 320a^2x^2 = -15x^2$

$320a^2 - 160a + 15 = 0$

$a = 1/8$ or $a = 3/8$

A Note from Mojza

This resource for AS Level Mathematics (9709) has been prepared by Team Mojza, covering the content for AS Level 2022-24 syllabus. The content of this resource has been prepared with utmost care. We apologise for any issues overlooked; factual, grammatical or otherwise. We hope that you benefit from these and find them useful towards achieving your goals for your Cambridge examinations.

If you find any issues within these notes or have any feedback, please contact us at support@mojza.org.

Acknowledgements

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