MOJZA

AS Level

Mechanics (M1)

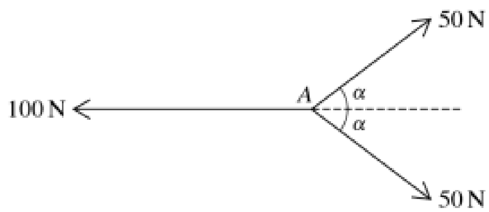
9709/04 Difficult Questions Compilation

BY TEAM MOJZA

CONTENTS

FORCES AND EQUILIBRIUM

9709_s20_qp_41



Three coplanar forces of magnitudes 100 N, 50 N and 50 N act at a point A, as shown in the diagram. The value of $\cos\alpha$ is $\frac{4}{5}$.

Find the magnitude of the resultant of the three forces and state its direction. [3]

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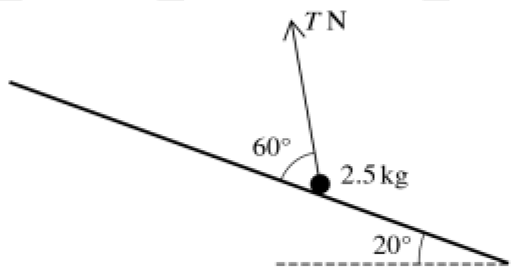
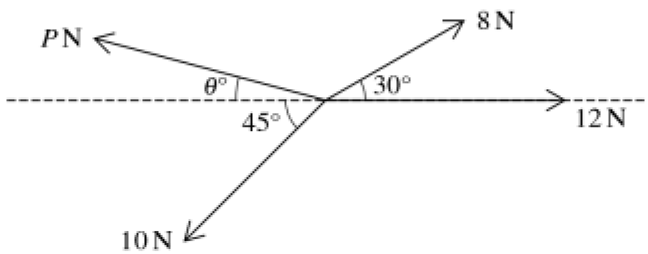


A particle of mass 2.5 kg is held in equilibrium on a rough plane inclined at 20° to the horizontal by a force of magnitude T N making an angle of 60° with a line of greatest slope of the plane (see diagram). The coefficient of friction between the particle and the plane is 0.3.

Find the greatest and least possible values of T. [8]

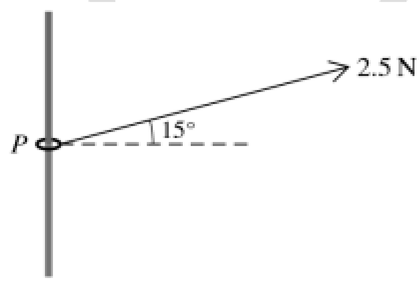
9709_W20_qp_41



Coplanar forces of magnitudes 8 N, 12 N, 10 N and P N act at a point in the directions shown in the diagram. The system is in equilibrium.

Find P and θ. [6]

9709_m19_qp_42



A small ring P of mass 0.03 kg is threaded on a rough vertical rod. A light inextensible string is attached to the ring and is pulled upwards at an angle of 15° to the horizontal. The tension in the string is 2.5 N (see diagram). The ring is in limiting equilibrium and on the point of sliding up the rod. Find the coefficient of friction between the ring and the rod. [4]

9709_w19_qp_43

A crate of mass 500 kg is being pulled along rough horizontal ground by a horizontal rope attached to a winch. The winch produces a constant pulling force of 2500 N and the crate is moving at constant speed. Find the coefficient of friction between the crate and the ground. [3]

9709_s22_qp_43

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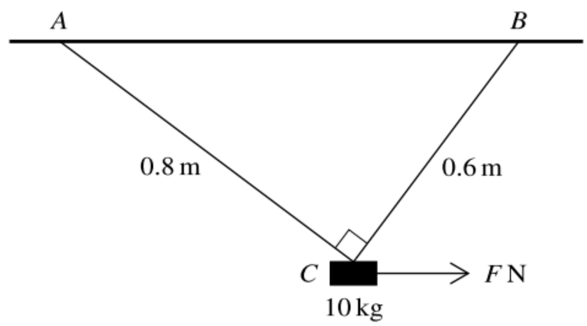

The diagram shows a block of mass 10 kg suspended below a horizontal ceiling by two strings AC and BC, of lengths 0.8 m and 0.6 m respectively, attached to fixed points on the ceiling. Angle $ACB = 90°$. There is a horizontal force of magnitude F N acting on the block. The block is in equilibrium.

(a) In the case where $F = 20$, find the tensions in each of the strings. [5]

(b) Find the greatest value of F for which the block remains in equilibrium in the position shown. [3]

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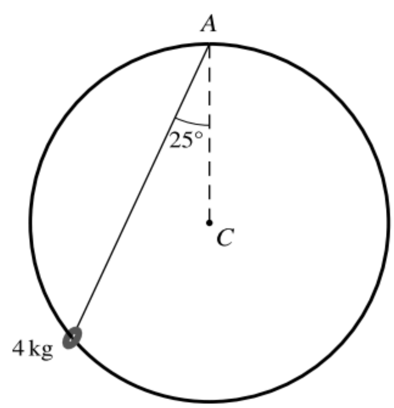

A ring of mass 4 kg is threaded on a smooth circular rigid wire with centre C. The wire is fixed in a vertical plane and the ring is kept at rest by a light string connected to A, the highest point of the circle. The string makes an angle of 25° to the vertical (see diagram).

Find the tension in the string and the magnitude of the normal reaction of the wire on the ring. [6]

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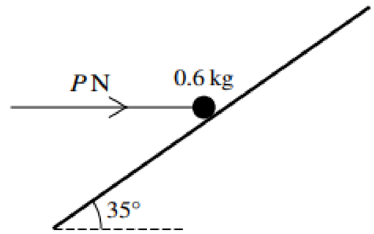

A particle of mass 0.6 kg is placed on a rough plane which is inclined at an angle of 35° to the horizontal. The particle is kept in equilibrium by a horizontal force of magnitude P N acting in a vertical plane containing a line of greatest slope (see diagram). The coefficient of friction between the particle and plane is 0.4.

Find the least possible value of P. [6]

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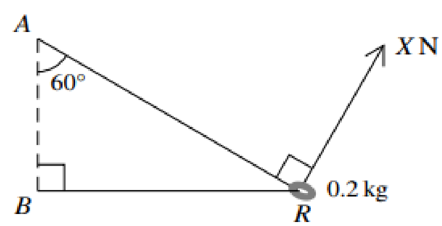

A smooth ring R of mass 0.2 kg is threaded on a light string ARB. The ends of the string are attached to fixed points A and B with A vertically above B. The string is taut and angle $ABR = 90°$. The angle between the part AR of the string and the vertical is 60°. The ring is held in equilibrium by a force of magnitude X N, acting on the ring in a direction perpendicular to AR (see diagram).

Calculate the tension in the string and the value of X. [5]

KINEMATICS

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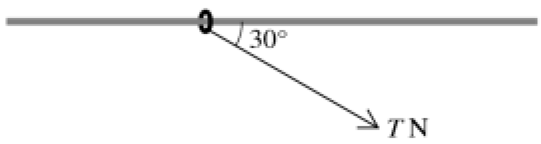

The diagram shows a ring of mass 0.1 kg threaded on a fixed horizontal rod. The rod is rough and the coefficient of friction between the ring and the rod is 0.8. A force of magnitude T N acts on the ring in a direction at 30° to the rod, downwards in the vertical plane containing the rod. Initially the ring is at rest.

(a) Find the greatest value of T for which the ring remains at rest. [4]

(b) Find the acceleration of the ring when $T = 3$. [3]

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A particle moves in a straight line AB. The velocity $v\,\mathrm{m\,s^{-1}}$ of the particle t s after leaving A is given by $v = k(t^2 - 10t + 21)$, where k is a constant. The displacement of the particle from A, in the direction towards B, is 2.85 m when $t = 3$ and is 2.4 m when $t = 6$.

(a) Find the value of k. Hence find an expression, in terms of t, for the displacement of the particle from A. [7]

(b) Find the displacement of the particle from A when its velocity is a minimum. [4]

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A particle travels in a straight line PQ. The velocity of the particle t s after leaving P is $v\,\mathrm{m\,s^{-1}}$, where

$$v = 4.5 + 4t - 0.5t^2.$$

(a) Find the velocity of the particle at the instant when its acceleration is zero. [3]

The particle comes to instantaneous rest at Q.

(b) Find the distance PQ. [6]

9709_W20_qp_41

A particle P moves in a straight line. It starts from rest at a point O on the line and at time t s after leaving O it has acceleration $a\,\text{m s}^{-2}$, where $a = 6t - 18$.

Find the distance P moves before it comes to instantaneous rest. [6]

9709_W20_qp_42

A particle is projected vertically upwards with speed $40\,\text{m s}^{-1}$ alongside a building of height h m.

(a) Given that the particle is above the level of the top of the building for 4 s, find h. [4]

(b) One second after the first particle is projected, a second particle is projected vertically upwards from the top of the building with speed $20\,\text{m s}^{-1}$.

Denoting the time after projection of the first particle by t s, find the value of t for which the two particles are at the same height above the ground. [4]

9709_s22_qp_41

6 A particle starts from a point O and moves in a straight line. The velocity $v\,\text{m s}^{-1}$ of the particle at time t s after leaving O is given by

$$v = k(3t^2 - 2t^3),$$

where k is a constant.

(a) Verify that the particle returns to O when $t = 2$. [4]

(b) It is given that the acceleration of the particle is $-13.5\,\text{m s}^{-2}$ for the positive value of t at which $v = 0$.

Find k and hence find the total distance travelled in the first two seconds of motion. [6]

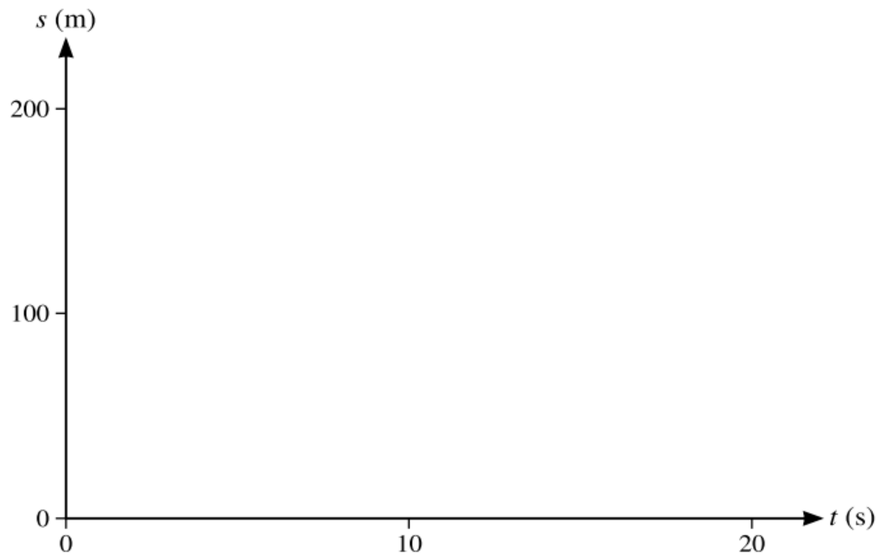
9709_s22_qp_42

4 A particle A, moving along a straight horizontal track with constant speed $8\,\text{m s}^{-1}$, passes a fixed point O. Four seconds later, another particle B passes O, moving along a parallel track in the same direction as A. Particle B has speed $20\,\text{m s}^{-1}$ when it passes O and has a constant deceleration of $2\,\text{m s}^{-2}$. B comes to rest when it returns to O.

(a) Find expressions, in terms of t, for the displacement from O of each particle t seconds after B passes O. [3]

(b) Find the values of t when the particles are the same distance from O. [3]

(c) On the given axes, sketch the displacement-time graphs for both particles, for values of t from 0 to 20. [3]



9709_W23_qp_41

7 A particle moves in a straight line starting from a point O before coming to instantaneous rest at a point X. At time t s after leaving O, the velocity $v\,\mathrm{m\,s^{-1}}$ of the particle is given by

$$v = 7.2t^2 \qquad 0 \leqslant t \leqslant 2,$$

$$v = 30.6 - 0.9t \qquad 2 \leqslant t \leqslant 8,$$

$$v = \frac{1600}{t^2} + kt \qquad 8 \leqslant t,$$

where k is a constant. It is given that there is no instantaneous change in velocity at $t = 8$.

Find the distance OX. [9]

MOMENTUM

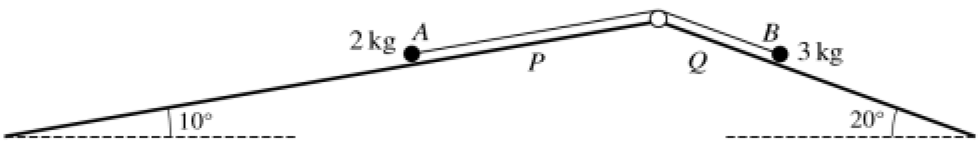
9709_s20_qp_43

Particles P of mass m kg and Q of mass 0.2 kg are free to move on a smooth horizontal plane. P is projected at a speed of $2\,m\,s^{-1}$ towards Q which is stationary. After the collision P and Q move in opposite directions with speeds of $0.5\,m\,s^{-1}$ and $1\,m\,s^{-1}$ respectively.

Find m. [3]

NEWTON'S LAWS OF MOTION

9709_W20_qp_43



As shown in the diagram, particles A and B of masses 2 kg and 3 kg respectively are attached to the ends of a light inextensible string. The string passes over a small fixed smooth pulley which is attached to the top of two inclined planes. Particle A is on plane P, which is inclined at an angle of 10° to the horizontal. Particle B is on plane Q, which is inclined at an angle of 20° to the horizontal. The string is taut, and the two parts of the string are parallel to lines of greatest slope of their respective planes.

(a) It is given that plane P is smooth, plane Q is rough, and the particles are in limiting equilibrium.

Find the coefficient of friction between particle B and plane Q. [5]

(b) It is given instead that both planes are smooth and that the particles are released from rest at the same horizontal level.

Find the time taken until the difference in the vertical height of the particles is 1 m. [You should assume that this occurs before A reaches the pulley or B reaches the bottom of plane Q.] [6]

9709_s19_qp_42

A constant resistance to motion of magnitude 350 N acts on a car of mass 1250 kg. The engine of the car exerts a constant driving force of 1200 N. The car travels along a road inclined at an angle of θ to the horizontal, where $\sin\theta = 0.05$. Find the speed of the car when it has moved 100 m from rest in each of the following cases.

- The car is moving up the hill.
- The car is moving down the hill. [7]

9709_m18_qp_42

Two particles A and B, of masses 0.8 kg and 0.2 kg respectively, are connected by a light inextensible string that passes over a fixed smooth pulley. The particles hang vertically. The system is released from rest. Show that the acceleration of A has magnitude $6\,m\,s^{-2}$ and find the tension in the string. [4]

9709_m18_qp_42

A particle of mass 12 kg is on a rough plane inclined at an angle of 25° to the horizontal. A force of magnitude P N acts on the particle. This force is horizontal and the particle is on the point of moving up a line of greatest slope of the plane. The coefficient of friction between the particle and the plane is 0.8. Find the value of P. [6]

9709_w18_qp_41

A particle of mass 0.2 kg moving in a straight line experiences a constant resistance force of 1.5 N. When the particle is moving at speed $2.5\,\mathrm{m\,s^{-1}}$, a constant force of magnitude F N is applied to it in the direction in which it is moving. Given that the speed of the particle 5 seconds later is $4.5\,\mathrm{m\,s^{-1}}$, find the value of F. [4]

9709_s22_qp_43

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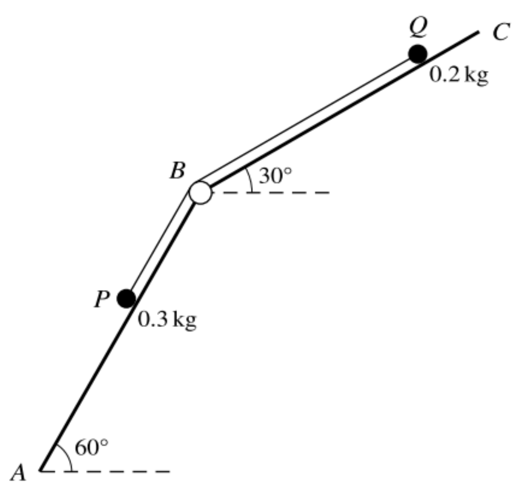

Two particles P and Q, of masses 0.3 kg and 0.2 kg respectively, are attached to the ends of a light inextensible string. The string passes over a fixed smooth pulley at B which is attached to two inclined planes. P lies on a smooth plane AB which is inclined at 60° to the horizontal. Q lies on a plane BC which is inclined at 30° to the horizontal. The string is taut and the particles can move on lines of greatest slope of the two planes (see diagram).

(a) It is given that the plane BC is smooth and that the particles are released from rest.

Find the tension in the string and the magnitude of the acceleration of the particles. [5]

9709_s23_qp_41

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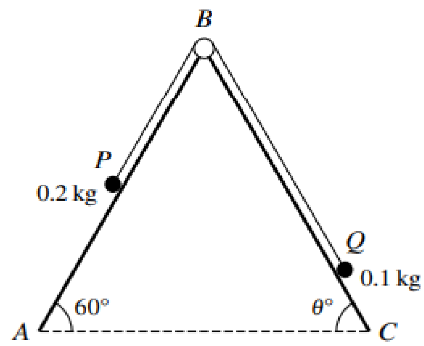

Two particles P and Q, of masses 0.2 kg and 0.1 kg respectively, are attached to the ends of a light inextensible string. The string passes over a fixed smooth pulley at B which is attached to two inclined planes. Particle P lies on a smooth plane AB which is inclined at 60° to the horizontal. Particle Q lies on a plane BC which is inclined at an angle of $\theta°$ to the horizontal. The string is taut and the particles can move on lines of greatest slope of the two planes (see diagram).

(a) It is given that $\theta = 60$, the plane BC is rough and the coefficient of friction between Q and the plane BC is 0.7. The particles are released from rest.

Determine whether the particles move. [4]

(b) It is given instead that the plane BC is smooth. The particles are released from rest and in the subsequent motion the tension in the string is $(\sqrt{3} - 1)$ N.

Find the magnitude of the acceleration of P as it moves on the plane, and find the value of θ. [4]

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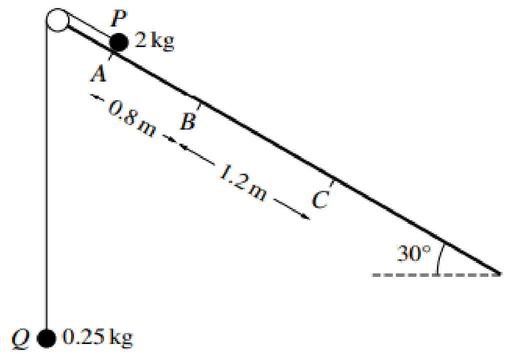

Two particles P and Q, of masses 2 kg and 0.25 kg respectively, are connected by a light inextensible string that passes over a fixed smooth pulley. Particle P is on an inclined plane at an angle of 30° to the horizontal. Particle Q hangs below the pulley. Three points A, B and C lie on a line of greatest slope of the plane with $AB = 0.8$ m and $BC = 1.2$ m (see diagram).

Particle P is released from rest at A with the string taut and slides down the plane. During the motion of P from A to C, Q does not reach the pulley. The part of the plane from A to B is rough, with coefficient of friction 0.3 between the plane and P. The part of the plane from B to C is smooth.

(a) **(i)** Find the acceleration of P between A and B. [4]

(ii) Hence, find the speed of P at C. [5]

(b) Find the time taken for P to travel from A to C. [4]

9709_W23_qp_42

6 A railway engine of mass 120 000 kg is towing a coach of mass 60 000 kg up a straight track inclined at an angle of α to the horizontal where $\sin\alpha = 0.02$. There is a light rigid coupling, parallel to the track, connecting the engine and coach. The driving force produced by the engine is 125 000 N and there are constant resistances to motion of 22 000 N on the engine and 13 000 N on the coach.

(a) Find the acceleration of the engine and find the tension in the coupling. [5]

At an instant when the engine is travelling at $30\,\text{m s}^{-1}$, it comes to a section of track inclined upwards at an angle β to the horizontal. The power produced by the engine is now 4 500 000 W and, as a result, the engine maintains a constant speed.

(b) Assuming that the resistance forces remain unchanged, find the value of β. [4]

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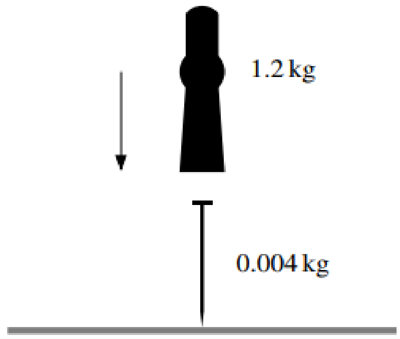
2



A machine for driving a nail into a block of wood causes a hammerhead to drop vertically onto the top of a nail. The mass of the hammerhead is 1.2 kg and the mass of the nail is 0.004 kg (see diagram). The hammerhead hits the nail with speed $v\,\text{m s}^{-1}$ and remains in contact with the nail after the impact. The combined hammerhead and nail move immediately after the impact with speed $40\,\text{m s}^{-1}$.

(a) Calculate v, giving your answer as an exact fraction. [2]

(b) The nail is driven 4 cm into the wood.

Find the constant force resisting the motion. [3]

ENERGY, WORK & POWER

9709_S22_qp_41

Car B starts off at the same instant as car A. The two cars arrive at P simultaneously and with the same speed. The engine of B produces a driving force of 3200 N and the car experiences a constant resistance to motion of 1200 N.

(b) Find the mass of B. [3]

(c) Find the steady speed which B can maintain when its engine is working at the same rate as it is at P. [3]

9709_W20_qp_41

A car of mass 1500 kg is pulling a trailer of mass 750 kg up a straight hill of length 800 m inclined at an angle of $\sin^{-1} 0.08$ to the horizontal. The resistances to the motion of the car and trailer are 400 N and 200 N respectively. The car and trailer are connected by a light rigid tow-bar. The car and trailer have speed $30\,\text{m s}^{-1}$ at the bottom of the hill and $20\,\text{m s}^{-1}$ at the top of the hill.

(a) Use an energy method to find the constant driving force as the car and trailer travel up the hill. [5]

After reaching the top of the hill the system consisting of the car and trailer travels along a straight level road. The driving force of the car's engine is 2400 N and the resistances to motion are unchanged.

(b) Find the acceleration of the system and the tension in the tow-bar. [4]

9709_W20_qp_43

Two small smooth spheres A and B, of equal radii and of masses 4 kg and m kg respectively, lie on a smooth horizontal plane. Initially, sphere B is at rest and A is moving towards B with speed $6\,\text{m s}^{-1}$. After the collision A moves with speed $1.5\,\text{m s}^{-1}$ and B moves with speed $3\,\text{m s}^{-1}$.

Find the two possible values of the loss of kinetic energy due to the collision. [6]

9709_m19_qp_42

A car of mass 1500 kg is pulling a trailer of mass 300 kg along a straight horizontal road at a constant speed of 20 m s^{-1}. The system of the car and trailer is modelled as two particles, connected by a light rigid horizontal rod. The power of the car's engine is 6000 W. There are constant resistances to motion of R N on the car and 80 N on the trailer.

(i) Find the value of R. [2]

The power of the car's engine is increased to 12 500 W. The resistance forces do not change.

(ii) Find the acceleration of the car and trailer and the tension in the rod at an instant when the speed of the car is 25 m s^{-1}. [5]

9709_s19_qp_41

A lorry has mass 12 000 kg.

(i) The lorry moves at a constant speed of 5 m s^{-1} up a hill inclined at an angle of θ to the horizontal, where $\sin\theta = 0.08$. At this speed, the magnitude of the resistance to motion on the lorry is 1500 N. Show that the power of the lorry's engine is 55.5 kW. [3]

When the speed of the lorry is v m s^{-1} the magnitude of the resistance to motion is kv^2 N, where k is a constant.

(ii) Show that $k = 60$. [1]

(iii) The lorry now moves at a constant speed on a straight level road. Given that its engine is still working at 55.5 kW, find the lorry's speed. [3]

9709_s19_qp_42

A particle of mass 13 kg is on a rough plane inclined at an angle of θ to the horizontal, where $\tan\theta = \frac{5}{12}$. The coefficient of friction between the particle and the plane is 0.3. A force of magnitude T N, acting parallel to a line of greatest slope, moves the particle a distance of 2.5 m up the plane at a constant speed. Find the work done by this force. [5]

9709_s22_qp_42

6 A car of mass 900 kg is moving up a hill inclined at $\sin^{-1} 0.12$ to the horizontal. The initial speed of the car is 11 m s^{-1}. After 12 s, the car has travelled 150 m up the hill and has speed 16 m s^{-1}. The engine of the car is working at a constant rate of 24 kW.

(a) Find the work done against the resistive forces during the 12 s. [5]

The car then travels along a straight horizontal road. There is a resistance to the motion of the car of $(1520 + 4v)$ N when the speed of the car is $v\text{ m s}^{-1}$. The car travels at a constant speed with the engine working at a constant rate of 32 kW.

(b) Find this speed. [3]

9709_s23_qp_43

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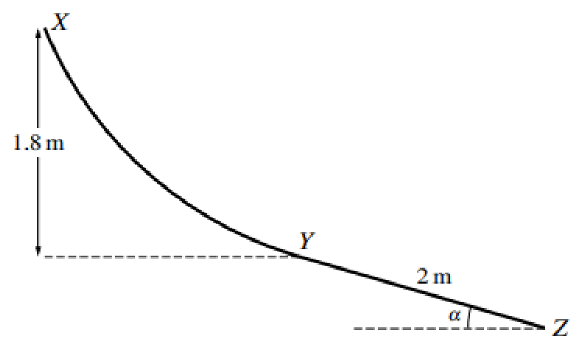

The diagram shows the vertical cross-section XYZ of a rough slide. The section YZ is a straight line of length 2 m inclined at an angle of α to the horizontal, where $\sin\alpha = 0.28$. The section YZ is tangential to the curved section XY at Y, and X is 1.8 m above the level of Y. A child of mass 25 kg slides down the slide, starting from rest at X. The work done by the child against the resistance force in moving from X to Y is 50 J.

(a) Find the speed of the child at Y. [4]

It is given that the child comes to rest at Z.

(b) Use an energy method to find the coefficient of friction between the child and YZ, giving your answer as a fraction in its simplest form. [6]

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A Note from Mojza

This resource for AS Level Mathematics (9709) has been prepared by Team Mojza, covering the content for AS Level 2022-24 syllabus. The content of this resource has been prepared with utmost care. We apologise for any issues overlooked; factual, grammatical or otherwise. We hope that you benefit from these and find them useful towards achieving your goals for your Cambridge examinations.

If you find any issues within these notes or have any feedback, please contact us at support@mojza.org.

Acknowledgements

Authors:
Muhammad Adil
Amna Mudassar

Proofreaders:
Musbah Moid
Aroosham Mujahid
Miraal Omer

Designers:
Fasiha Raza